

Cyclic Direct Simple Shear Test on Soft Clay at Low Normal Stress

As applicable to offshore pipeline axial walking

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*To my mother and father,
who are devoted like Eastern parents and liberal like Western parents*

Executive summary

Offshore pipelines play a significant role in transporting energy resources such as crude oil and natural gas from offshore platforms to processing facilities. The on-bottom stability of offshore pipelines is influenced significantly by the geotechnical conditions on the seabed. Pipelines undergo a number of thermal cycles during their operational life. At the end of each thermal cycle, some part of the expansion recovers, whereas the irrecoverable expansion accumulates at the free ends and causes the pipe to move axially in one direction, a phenomenon known as axial walking. The test results from the Monash Advanced Pipe testing System (MAPS) imply that pipe axial walking induces relative movements of the soil below the pipe. Since the pipe-soil interaction is extensively influenced by the soil response, the portion of the soil below the pipe which undergoes shearing, characterised as the shear zone, is significant for pipe axial walking assessment and thus needs thorough investigation.

The research program reported here investigates the behaviour of soil within the shear zone in pipe axial walking problems. Cyclic direct simple shear tests on soft clay at low normal stress are performed as applicable to axial walking problems. The soil response in the shear zone is characterised as undrained, partially drained, or drained, based on cyclic shearing velocities and the relationship between residual shear resistance and shearing velocity is investigated. Finite element analyses are also conducted to capture the behaviour of soil within the shear zone, utilising an advanced constitutive soil model. Based on the results from both experimental work and numerical analysis, a set of data is established which can be applied in the calibration of large-scale pipe axial walking modelling, and provide guidance on the design practice of offshore pipelines when considering on-bottom stability in axial direction.

Declaration

I hereby declare that material contained in this thesis has not been published in any other degree or diploma in any other institution. To the best of my knowledge no material presented within has been previously published or written by others except where references are sighted in the text.



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Chapter 1 - Introduction

1.1 Background

Offshore pipelines play a significant role in transporting energy resources such as crude oil and natural gas from offshore platforms to processing facilities. Due to the increasing demand for fossil fuels in recent years, and the development of technology which makes access to deep-sea energy resources possible, the offshore energy industry has grown rapidly. Longer pipelines are laid and such pipelines are required to operate at more extreme conditions. For instance, high operating temperature and pressure need to be applied within some offshore pipelines to prevent petroleum from solidification. Such new technical challenges mean that a better understanding of the behaviour of offshore pipelines is required and the design code for offshore pipelines needs to be upgraded accordingly.

In deep water, pipelines are often left unburied on the seabed. Therefore, the on-bottom stability of offshore pipelines is influenced significantly by the geotechnical conditions on the seabed. Pipeline instability may be caused by the loading from the wave-induced currents (Fujiwara et al., 2011, Myrhaug and Ong, 2011, Chakkarapani et al., 2011). The thermal expansion and contraction corresponding to the production and shutdown cycles of offshore platforms may also cause pipeline instability (Casola et al., 2011). Pipelines undergo a number of such thermal cycles during their operational life. At the end of each thermal cycle, some part of the expansion recovers, whereas the irrecoverable expansion accumulates at the free ends and causes the pipe to move axially in one direction, a phenomenon known as *axial walking*. Consequently, the soil in contact with the pipe is also subjected to both static and dynamic loading, which may affect the long-term mechanical stability of pipelines. The pipe-soil interaction is analogous to structure-soil interaction in foundation problems involving cohesive soils. However, unlike foundations of many structures where flexibility is usually not allowed, offshore pipelines can retain certain flexibility without exceeding the limit state at the centre (Randolph, 2011). Similar to foundation problems, the pipe-soil interaction can be divided into vertical, axial and lateral directions. The pipe-soil interaction in both vertical and lateral directions has been investigated extensively (Merifield et al., 2008, Zhou et al., 2008), whereas the axial pipe-soil interaction, which is regarded as the primary cause of pipeline instability, is yet to be clearly established. Some work has been done, for example, by Bruton et al. (2007), White et al. (2011a) and Randolph (2012). Most recently, a

specialised 2-D electric actuator system, the Monash Advanced Pipe-testing System (MAPS), has been developed at Monash University, Australia. The results from scaled axial soil-pipe interaction tests using MAPS have been made available (Senthilkumar, 2013).

The MAPS results imply that pipe axial walking induces relative movements of the soil below the pipe (Senthilkumar, 2013). Since the pipe-soil interaction is extensively influenced by the soil response, the portion of the soil below the pipe which undergoes shearing, characterised as the *shear zone*, is significant for pipe axial walking assessment, and therefore needs to be investigated thoroughly. Cyclic direct simple shear testing is proposed as the preferred testing method to conduct the investigation owing to its various merits, including its loading pattern, allowing the rotation of principal stress axes, the small specimen size required and its shear failure planes.

The results of the current research program will advance the understanding of soil behaviour within the shear zone in pipeline axial walking problems, by performing laboratory soil tests and conducting numerical modelling and analyses. Based on the results from both experimental work and numerical analysis, a set of data will be established which can be applied in the calibration of large-scale pipe axial walking modelling, and provide guidance on the design practice of offshore pipelines when considering on-bottom stability in axial direction.

1.2 Research objectives

The aim of the research program is to investigate soil behaviour within the shear zone in pipe axial walking problems, and to advance the understanding of offshore pipeline instability in the axial direction.

The specific objectives are:

1. Provide a comprehensive literature review to build a conceptual framework upon which the research program is based.
2. Establish an appropriate laboratory testing approach to investigate the soil behaviour within the shear zone.
3. Develop numerical modelling capacity to capture the soil behaviour within the shear zone in finite element analysis.
4. Based on the results from experimental work and numerical analysis, complement and improve the results obtained from MAPS tests.
5. Establish a set of data which can be applied in the calibration of the modelling of large-scale pipe axial walking.

1.3 Structure of thesis

This thesis consists of five chapters, a list of references and appendices. The chapters are summarised below.

Chapter 1 – Introduction

The first chapter contains an introduction to the problem, the scope of the project and the objectives.

Chapter 2 – Literature review

Chapter 2 provides a comprehensive literature review upon which the conceptual framework is built.

Chapter 3 – Experimental program and results

Chapter 3 presents the experimental program and the experimental results. First the soil selected for the experimental program is introduced. This is followed by an elaboration of the experimental set-up and the presentation of the experimental results.

Chapter 4 – Numerical modelling

The numerical modelling work of the current research program is presented in this chapter. The critical state soil theory, upon which the Modified Cam-clay model is based, and the model itself are introduced, followed by the two-dimensional finite element analysis (FEA) of the cyclic direct simple shearing on soft clay. Finally the results of the analyses are presented.

Chapter 5 – Conclusions and recommendations

The major findings of the research program are summarised and conclusions are presented, followed by recommendations for future research.

Appendices

Detailed elaborations of the critical state soil theory and the modified Cam-clay model are provided in Appendix A. Additional finite element analysis results are presented in Appendix

B. An example “.inp” file for the two-dimensional finite element soil model developed is provided in Appendix C

Chapter 2 - Literature review

A comprehensive literature review was conducted to build a conceptual framework upon which the research program was based. First, current research into offshore pipeline axial walking is introduced. An investigation of the shear influence zone induced by axially-walking offshore pipelines is presented. A generic pipe load-displacement relationship that can be applied to characterise pipe axial walking behaviour is discussed. This is followed by the proposal to use cyclic direct simple shear testing to investigate the soil behaviour within the shear zone. A discussion of the relevant parameters in offshore pipeline axial walking is presented. Detailed information about direct simple shear testing is introduced in terms of its development, and variety and uniformity in soil specimens.

2.1 Offshore pipeline axial walking

Offshore pipeline axial walking is a phenomenon of the undesired accumulation of axial displacement of offshore pipelines during operational cycles. Tornes et al. (2000) provided the first comprehensive study of pipe axial walking, and this was followed by analytical solutions linking pipe-soil friction and pipe axial walking presented by Carr et al. (2006). White et al. (2011a) reported some experimental data on pipe-soil axial resistance, demonstrating the influence of drainage and excess pore water pressure. The data indicate that axial resistance, as a portion of submerged pipe weight, can vary from as low as 0.1 to greater than 1. A framework for pipe-soil axial resistance based on an effective stress approach was set out by White & Cathie (2011), covering both undrained and drained conditions. It was argued that axial resistance tends towards the drained values during cycles of pipe movement, regardless of the rate or duration of each movement. This is due to the fact that the soil surrounding the pipe will eventually reach a critical state, at which excess pore water pressure generation will not take place. A reliable prediction of this mechanism could bring significant design benefits, as drained resistance is usually higher than undrained resistance. Consequently, a higher range of axial resistance can be applied, leading to more cost-effective design.

2.1.1 Axial pipe-soil interaction

Bruton et al. (2008) divided axial pipe-soil interaction into two stages as shown by the ‘brittle’ response in Figure 2.1: (i) breakout axial resistance and (ii) residual axial resistance. When

the pipe is loaded for the first time, a breakout or peak axial resistance can take place which reduces to a residual axial resistance after breakout. The displacement at which this peak occurs is defined as the mobilisation displacement. A peak is not observed during subsequent loading and this leads to a ductile breakout response, as illustrated by the ‘ductile’ curve in Figure 2.1.

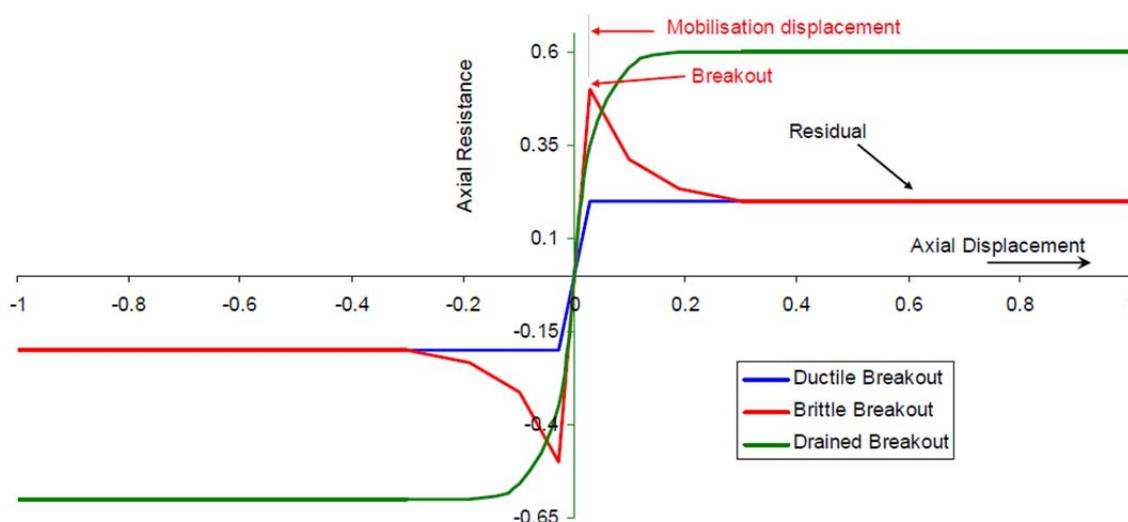


Figure 2.1 Axial friction resistance schematic with mobilisation displacement and breakout (Bruton et al., 2008)

The friction falls away to residual axial resistance when the pipeline reaches larger axial displacement, and the term ‘residual’ is used by analogy with the residual friction angle which is mobilised within fine-grained soil after continued shearing along a single plane.

The ‘drained’ response shown in Figure 2.1 corresponds to slow pipe axial displacement, where excess pore water pressure does not build up, and the axial resistance is profoundly higher in this case.

The axial response is thus significantly influenced by the generation and dissipation of excess pore water pressure which leads to undrained and drained soil behaviour. It is likely that the response is between fully undrained and fully drained in typical field conditions.

2.1.2 Drained and undrained soil resistance

Oliphant and Maconochie (2006) presented a method for estimating axial soil resistance using a relative roughness parameter, and made recommendations on the displacements

necessary to mobilise peak and residual undrained and drained soil resistance. It is argued that the application of undrained and drained soil resistance will depend on the rate of straining as well as on the duration of pipeline loading for axial resistance.

The dimensionless group vD/c_v can be used to establish the most likely soil response (Randolph and House, 2001), where v is the velocity of the pipeline, D is the pipe diameter and c_v is the consolidation coefficient. The soil response may be bounded by

$$\frac{vD}{c_v} < 1.0 \text{ for the fully drained condition and} \quad (2.1)$$

$$\frac{vD}{c_v} > 20.0 \text{ for the fully undrained condition} \quad (2.2)$$

The value of c_v depends on both the permeability and compressibility of the soil and should be measured at low stress levels, as applicable to offshore pipeline axial walking.

During interface shearing, the response of a fine-grained soil ranges from fully drained to fully undrained for various velocities (White and Cathie, 2011). Figure 2.2 demonstrates the steady residual resistance measured during monotonic shearing of normally consolidated kaolin clay over a rough steel surface at different velocities. The kaolin was normally consolidated to 2.5kPa before shearing. A direct shear box was modified to operate at low stress levels as applicable to the offshore pipeline context. The results from shearing of a concrete-clay till interface at various velocities (Steenfelt, 1993) are plotted in Figure 2.2.

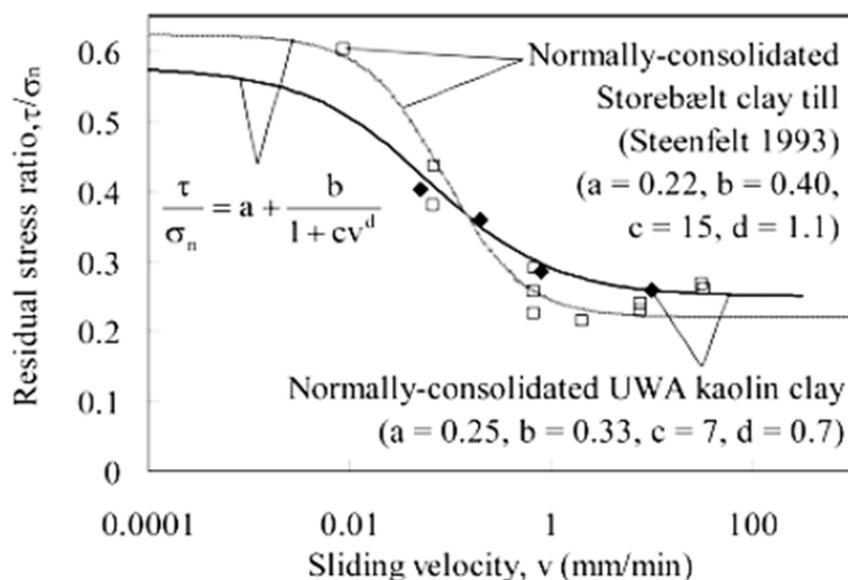


Figure 2.2 Interface shearing: kaolin clay and Storeælt clay till (White and Cathie, 2011, Steenfelt, 1993)

The responses of both the kaolin clay and the Storeælt clay till are close in terms of both the limiting drained and undrained resistances as well as the velocity range over which the transition takes place.

2.1.3 Cyclic axial response in an effective stress framework

The first deep water deployment of the Fugro SMARTPIPE (White et al., 2011b) focused on the axial pipe-soil interaction on soft clay. The tool was equipped with pore pressure measurement capacity on the surface of the test pipe, which allows the cyclic axial response to be interpreted in an effective stress framework.

The time histories of imposed vertical load, imposed axial pipe movement and measured axial resistance are presented in Figure 2.3. A modest peak in axial resistance was observed during the first sweep and ductile response followed during all subsequent sweeps.

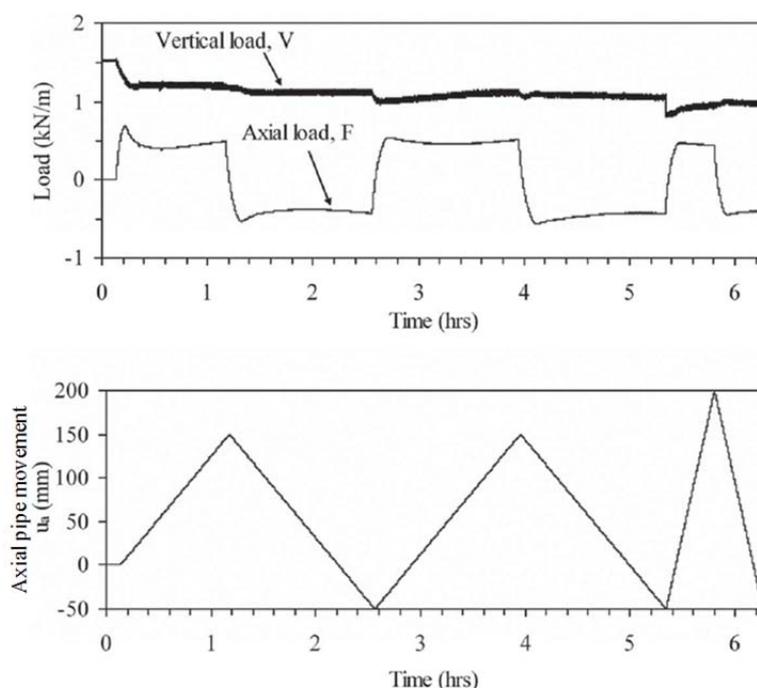


Figure 2.3 Time history during cyclic axial pipe testing (White et al., 2011b)

As it is possible to measure the excess pore pressure at the pipe soil surface, the data obtained from the cyclic axial pipe test can be compared in terms of both total stress and effective stresses. The mean shear stress τ_{av} on the pipe surface is calculated as

$$\tau_{av} = \frac{F}{\pi D/2} \quad (2.3)$$

where F is the axial pipe-soil load per unit length and D is diameter of the model pipe.

The friction factor based on the total stress FF is defined as

$$FF = \tau_{av}/V \quad (2.4)$$

where V is the vertical pipe-soil load per unit length

The friction factor based on effective stress FF' is defined as

$$FF' = \tau_{av}/\sigma'_{n,av} \quad (2.5)$$

where $\sigma'_{n,av}$ is the mean normal or vertical effective stress

The friction factor response during cyclic axial pipe test is shown in Figure 2.4, both in terms of total stress and effective stress. The total stress response is smoother than the effective

stress response. This is due to the variations between the excess pore pressures measured by the sensors, which indicate that the excess pore pressure was not uniformly distributed at the pipe soil surface.

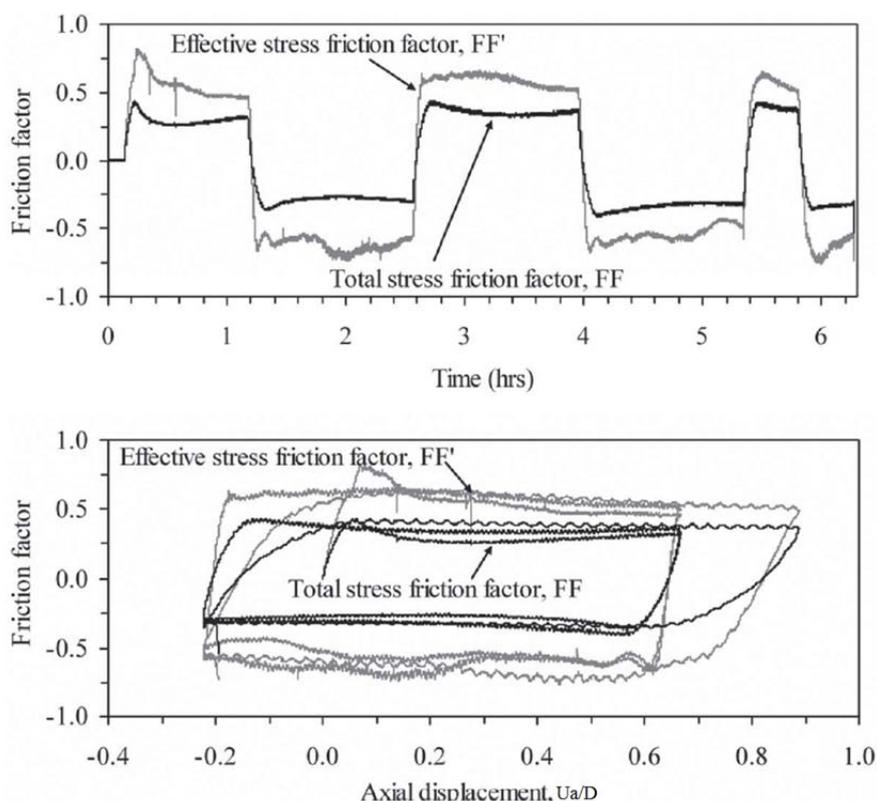


Figure 2.4 Friction factor response in cyclic axial pipe testing (White et al., 2011b)

When the responses are plotted in the form of total and effective stress failure criteria as illustrated in Figure 2.5, the effective stress interpretation is the most consistent. While the axial response in total stress interpretation varies for each cycle, all sweeps in each direction overlie each other in the effective stress interpretation.

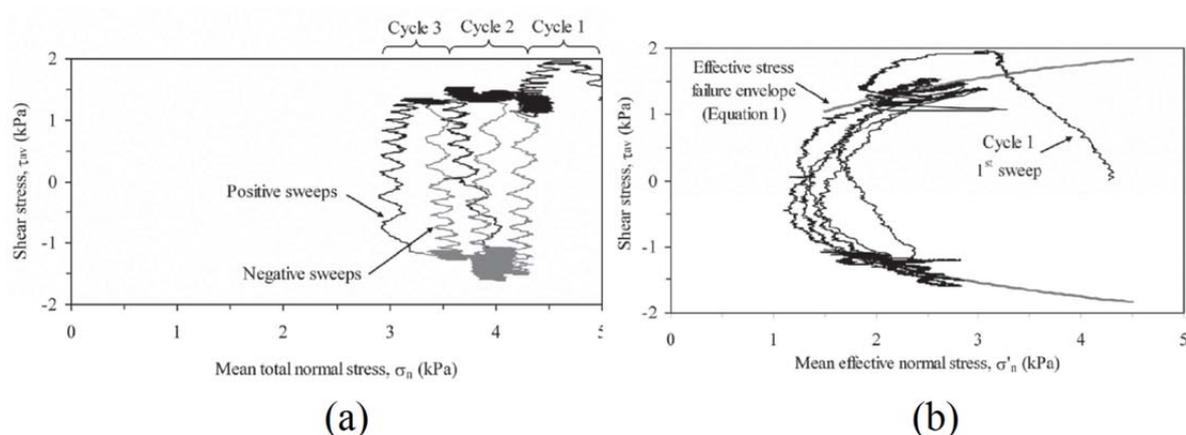


Figure 2.5 (a) Total stress interpretation of axial response (b) Effective stress interpretation of axial response (White et al., 2011b)

Analysis of the field data also suggests that the mobilisation of axial resistance is a time-related process instead of being linked to the distance of shearing. The suggestion is consistent with the excess pore pressure development in the sweeps. In other words, full pipe-soil resistance is only mobilised when steady pore pressures are reached.

2.1.4 Theoretical framework for axial resistance

Randolph et al. (2012) introduced a theoretical framework based on planar shearing to assess the magnitude of axial resistance. In the scenario shown in Figure 2.6(a), a slab is sheared at velocity v across a soil surface. The pressure exerted by the slab on the soil is q , and shearing occurs within a shear band of thickness h_s . Excess pore pressure Δu_s is generated during shearing and a shear strain of γ is reached after a time $t = \gamma h_s/v$.

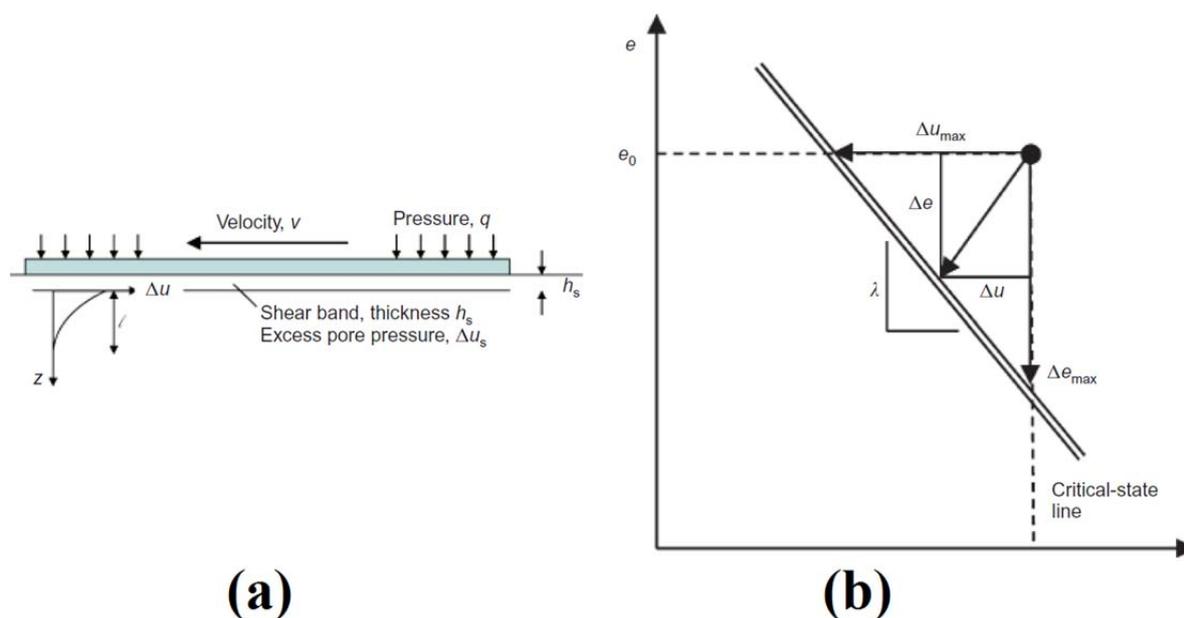


Figure 2.6 (a) Schematic diagram for analysis of velocity effects for planar shearing (b) Stress paths in e - $\ln\sigma_v$ space (Randolph et al., 2012)

The response of soil within the shear band is presented in Figure 2.6(b). Compression takes place in drained shearing corresponding to an ultimate decrease in void ratio of Δe_{max} , and the volumetric strain in this case is $\Delta\varepsilon_{v,max}$. For undrained shearing, excess pore pressure of Δu_{max} is generated. When the shearing is partially drained or partially undrained, the volumetric strain $\Delta\varepsilon_v$ and excess pore pressure Δu may be related by

$$\begin{aligned}\Delta\varepsilon_v &= -\frac{\Delta e}{1+e_0} \\ &= \Delta\varepsilon_{v,max} + \frac{\lambda}{1+e_0} \ln\left(1 - \frac{\Delta u}{q}\right)\end{aligned}\quad (2.6)$$

The corresponding limiting shear stress, τ_f , can be expressed as

$$\begin{aligned}\tau_f &= (q - \Delta u)\tan\phi' \\ &= \mu q\left(1 - \frac{\Delta u}{q}\right)\end{aligned}\quad (2.7)$$

where $\mu = t \tan\phi'$

The zone influenced by excess pore water pressures may be considered to extend a distance of l into the soil, where

$$l = \sqrt{12c_v t} \quad (2.8)$$

Therefore the volumetric strain $\Delta\varepsilon_v$ can be expressed as

$$\begin{aligned} \Delta\varepsilon_v &= \frac{\lambda}{1+e_0} \frac{\Delta u}{q} \sqrt{1.33 \frac{c_v t}{h_s^2}} \\ &= \frac{\lambda}{1+e_0} \frac{\Delta u}{q} \sqrt{1.33 \gamma \frac{c_v}{h_s v}} \end{aligned} \quad (2.9)$$

Combine this relationship with Equation 2.1 gives

$$\begin{aligned} \Delta\varepsilon_{v,max} \frac{1+e_0}{\lambda} + \ln\left(1 - \frac{\Delta u}{q}\right) &= \frac{\Delta u}{q} \sqrt{1.33 \frac{c_v t}{h_s^2}} \\ &= \frac{\Delta u}{q} \sqrt{1.33 \gamma \frac{c_v}{h_s v}} \end{aligned} \quad (2.10)$$

Figure 2.7(a) provides example relationships between excess pore pressure and time or shear strain divided by normalised velocity vh_s/c_v . Four curves for various values of $\Delta\varepsilon_{v,max}(1+e_0)/\lambda$ are presented. $\Delta\varepsilon_{v,max}(1+e_0)/\lambda$ is a quantity which quantifies the distance of the initial state from the critical state. The upper limit of $\Delta u/q$ is close to unity, resulting in almost zero effective stress on shearing.

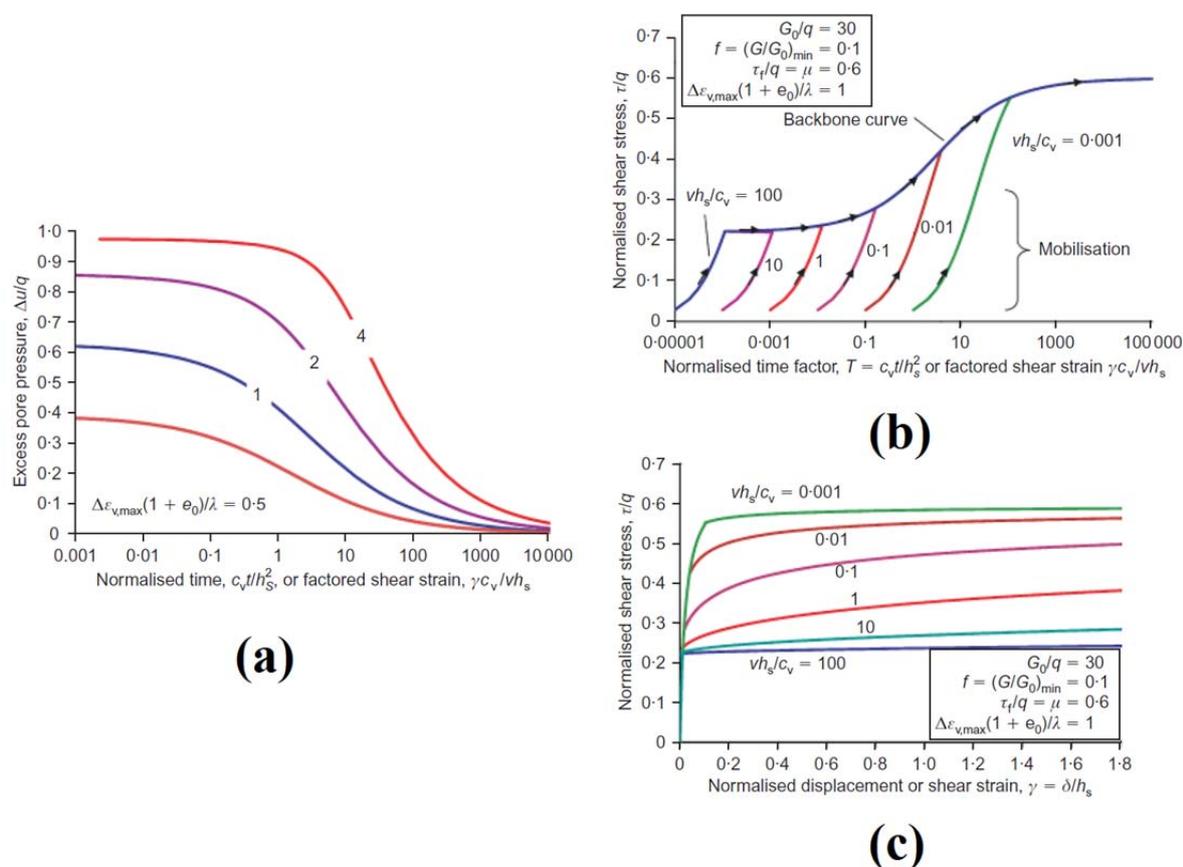


Figure 2.7 (a) Example relationships between excess pore pressure and time or shear strain Planar shearing responses for different normalised velocities Variation in resistance with (b)time; (c)displacement (Randolph et al., 2012)

To develop a complete shear stress-displacement relationship, Randolph et al. (2012) proposed a simple hyperbolic response of the form

$$\tau = \frac{G_0 \gamma}{1 + R_f G_0 \gamma / \tau_f} \quad (2.11)$$

with a small-strain shear modulus G_0 and hyperbolic parameter R_f (Duncan and Chang, 1970).

Figure 2.7(b) shows the variation of τ/q with time for different normalised velocities, vh_s/c_v . The responses for various velocities are intercepted by a common backbone curve after failure based on Equation (2.7). The same responses are shown in Figure 2.7(c) as a function of displacement, normalised by the shear band thickness h_s .

2.2 Shear zone in offshore pipeline axial walking

Axially-walking offshore pipelines induce relative movement of the soil underneath the pipe. Since the pipe-soil interaction is extensively influenced by the soil response, the portion of the soil below the pipe which undergoes shearing, characterised as the *shear zone*, is significant for pipe axial walking assessment. To characterise the shear zone, Senthilkumar (2013) applied image correlation analysis in MAPS testing. Based on the observations and results from the experiments conducted, the shear zone can be illustrated as in Figure 2.8.

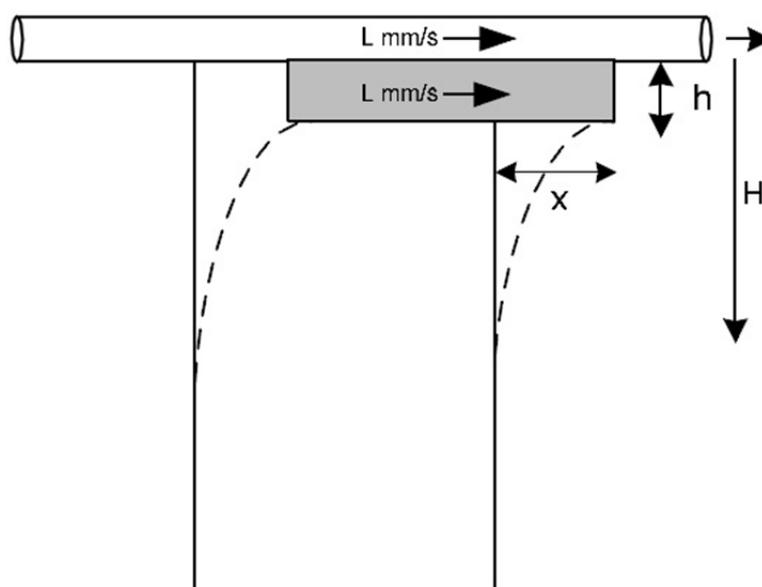


Figure 2.8 Shear zone underneath axially-walking pipe (Senthilkumar, 2013)

The shear zone consists of two parts. The first part, *zone h*, is the soil block from the depth immediately below the pipe to a depth of h . When pipe axial walking takes place, the soil within zone h moves together with the pipe at the same displacement rate. This zone can be considered as the direct failure zone. The second part, *zone H-h*, is the soil block from the depth of h to the depth of H , where H is the limit depth of shear influence. The soil in this zone is mobilised when axial walking occurs, but the soil does not move together with the pipe at the same displacement rate. Instead, the displacement decreases as the depth increases.

The experimental data (Senthilkumar, 2013) suggest that higher axial displacement rate would lead to a deeper shear zone, i.e. a larger value of H . It was found that the depth of zone h increases with both pipe axial displacement rate and pipe embedment. In contrast, the depth of zone $H-h$ increases with increasing axial displacement rate, but it is largely independent of

pipe embedment. The axial displacement rate depends primarily on soil axial resistance as well as on the expansion/contraction rate of the pipe wall, which in turn depends on the heating/cooling rate and the properties of the pipe and the soil. If the pipe axial displacement rate can be reduced, the extent of the shear influence on soil can also be reduced.

It is also worth noticing that the soil within the shear zone is subjected to axial loading while under vertical loading due to the pipe's weight. This loading pattern can be closely resembled by the direct simple shear test, as shown in Figure 2.9, in particular for *zone H-h*.

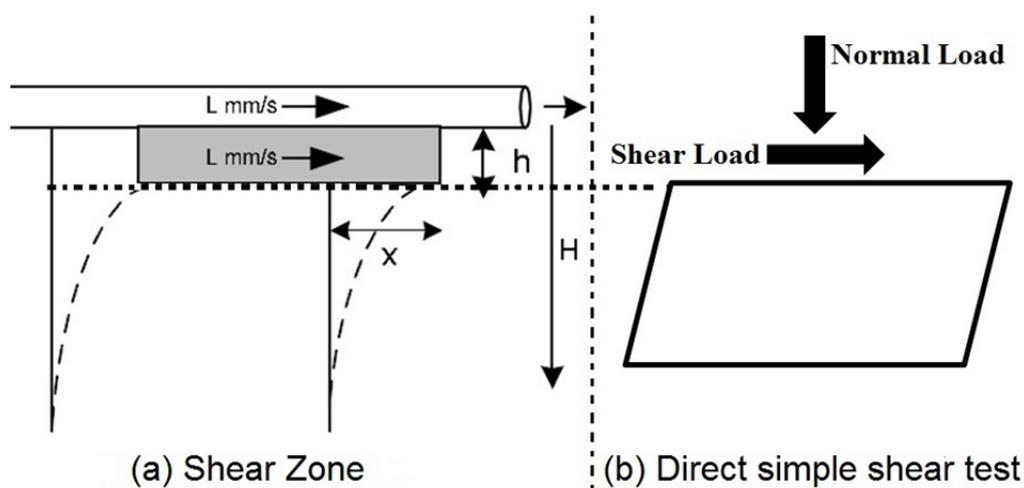


Figure 2.9 (a) Shear zone underneath axially walking pipe and (b) loading pattern of direct simple shear test

2.3 Generic load-displacement relationship in pipe axial walking

(Senthilkumar, 2013) proposed a generic pipe load-displacement relationship curve (Figure 2.10), which can be applied to characterise pipe axial walking behaviour. This load-displacement curve was constructed based on the experimental outcomes of physical pipe-soil interaction testing, using the Monash Advanced Pipe-testing System (MAPS).

The load-displacement curve comprises three major components: pre-peak, peak and residual resistance. The experimental results show that at a particular embedment depth, the rate of pipe axial displacement has no or negligible influence on the pre-peak load-displacement relationship. Therefore, in Figure 2.10 the pre-peak portions of the fast/undrained and

slow/draind load-displacement curves are the same for a particular embedment depth. However, increasing embedment depth means an increase in contact area, resulting in higher axial resistance.

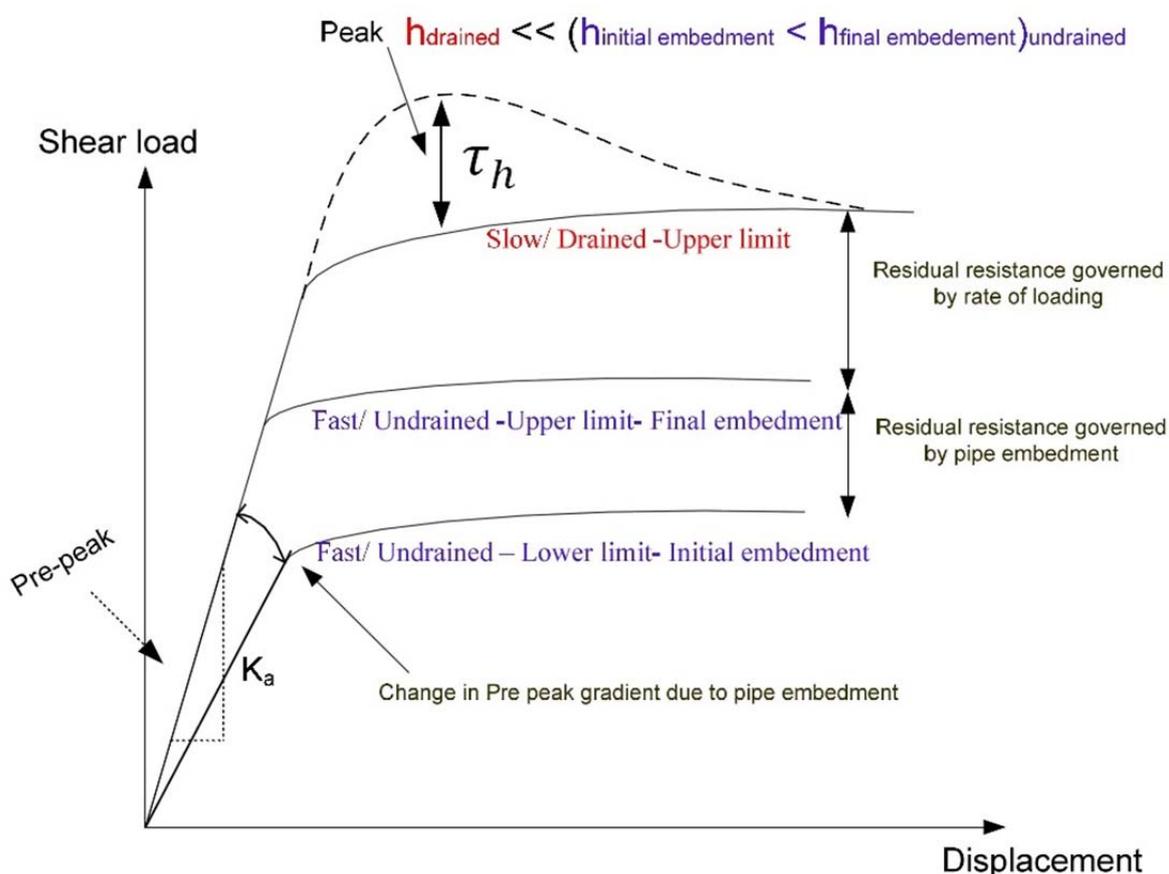


Figure 2.10 Generic pipe load-displacement curve with emphasis on residual resistance (Senthilkumar, 2013)

The peak resistance of the shear load, depicted by τ_h in Figure 2.10, was found at its highest value in the initial shearing cycle based on experimental observation. With increasing number of shearing cycles τ_h weakens gradually and eventually becomes negligible. The peak resistance τ_h was influenced by the embedment of the pipe and was found to be more profound in undrained conditions than in drained conditions.

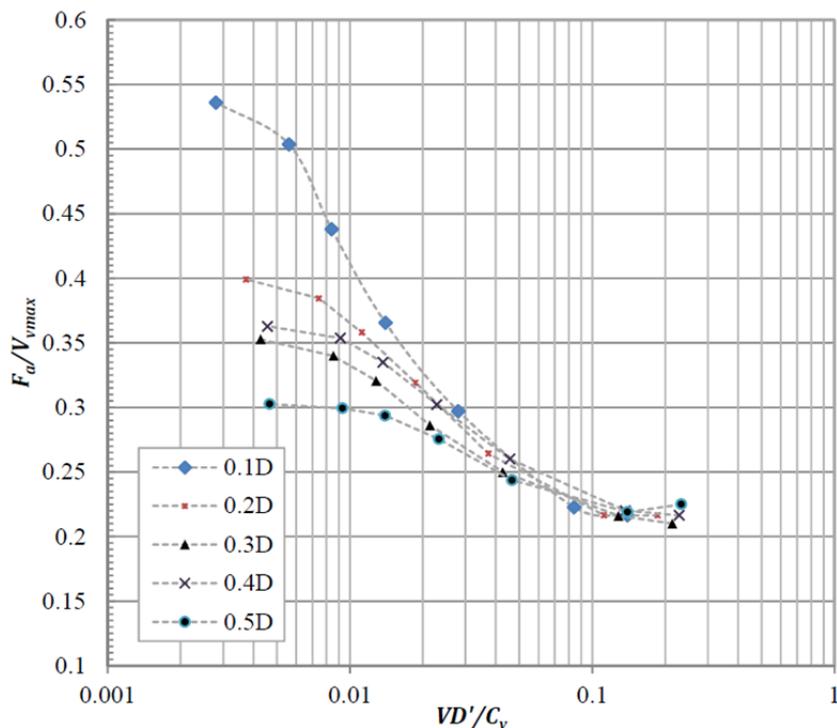
The residual resistance, which is the primary component in the load-displacement curve, was dominated by pipe axial displacement rate. The fast/undrained residual resistance is defined as the undrained limit, whereas the slow/drained one is defined as the drained limit. The region between the undrained and drained limits is regarded as the transition zone and

characterised as *the failure region governed by pipe axial displacement rate*. Furthermore, under undrained conditions, the level of residual resistance was also found to be influenced by pipe embedment. Therefore, the undrained limit of residual resistance is further divided into the upper and lower undrained limits, based on pipe embedment. The region between the lower and upper undrained limits is characterised as *the failure region governed by pipe embedment*.

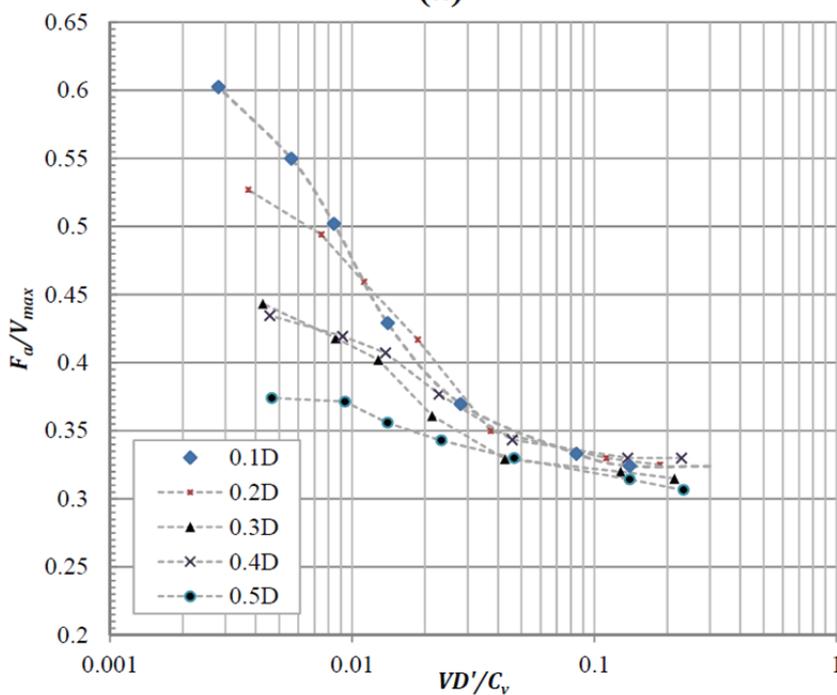
2.4 Residual resistance and undrained/drained limits in MAPS

The residual resistance levels in MAPS tests at various displacement rates are presented in Figure 2.11 (Senthilkumar, 2013). The axial pipe residual resistance is normalised by the maximum recorded vertical load V_{max} and the axial displacement rate is normalised by the effective pipe diameter D' and the coefficient of consolidation C_v . Figure 2.11(a) shows the residual resistance for smooth pipe surface conditions and Figure 2.11(b) shows the residual resistance for rough pipe surface conditions.

The plots show that the residual resistances at different pipe embedments converge to an unchanged level when the axial displacement rate increases. This limit can be considered as the axial displacement rate beyond which the residual resistance is not affected by the increase of axial displacement rate. In other words, it can be considered as the undrained limit. Similarly, the drained limit of the residual resistance can be obtained from the plots, and it is observed that higher embedment depths result in higher level of residual resistance.



(a)



(b)

Figure 2.11 Normalised residual resistance at various displacement rates and pipe embedments for (a) smooth pipe and (b) rough pipe (Senthilkumar, 2013)

2.5 Utilising cyclic direct simple shear test in offshore pipeline axial walking problems

In offshore pipeline axial walking problems, the seabed soil underneath the pipe undergoes cyclic shearing with low vertical/normal stress imposed. In order to further investigate the soil behaviour within the shear zone, in particular the load-displacement relationship, cyclic direct simple shear testing was proposed as the preferred testing method. Direct simple shear testing has been used extensively in offshore geotechnical engineering applications. The increasing popularity of the direct simple shear test is mainly due to the relatively small sample size required, and the shearing pattern, which closely resembles the loading condition in offshore geotechnical engineering problems. Particularly for offshore pipeline axial walking scenarios, the soil within the shear zone is subjected to horizontal cyclic shearing while under a vertical load, which is similar to the loading pattern in the direct simple shear test. Furthermore, cyclic shearing can be applied in the direct simple shear test, which can simulate the expansion and contraction in pipeline operation and shutdown cycles. The direct simple shear test also has merits compared with other testing methods. It is the only standard test that allows the rotations of principle stress axes (Airey and Wood, 1987) and unlike in direct shear testing where the shear failure plane is fixed as the horizontal plane, the failure plane in direct simple shear tests is not pre-defined.

When utilising direct simple shear tests in investigating pipeline axial walking problems, it is important to have sound knowledge of the relevant parameters involved. It has been determined that the primary variables are effective normal stress, shearing velocity and shearing amplitude. Some research literature has given guidance on the magnitude of these variables.

Normally the magnitude of the effective normal stress imposed on the soil underneath the pipe is relatively low in pipe axial walking problems. The effective normal stress (σ'_n) equals the total normal stress (σ_n) minus the pore water pressure (u). In drained cases, total normal stress (σ_n) may not be equal to the effective normal stress (σ'_n) since pore water pressure can be very high due to the depth of seabed, which can be in the order of kilometres. Bruton et al. (2007) suggest that the effective normal stress generated by typical pipeline weights is at the levels of 2kPa to 10kPa. The first stage of the Furgo SMARTPIPE project (White et al.,

2011b) saw an effective normal stress between 1kPa to 3kPa. In a pipe interface shearing example given by White and Cathie (2011), an effective normal stress of 5kPa was applied.

Regarding the shearing velocities, the magnitude of pipeline velocities ranges from 0.5mm/s to 0.001mm/s, and could even be lower for fully drained conditions. Bruton et al. (2007) advised that the available field data on pipeline velocities were limited, and an example problem provided in the literature showed that pipeline velocities only exceed 0.2mm/s near pipe ends and the durations are very short, and an average velocity of 0.005mm/s was given. In the first stage of the Furgo SMARTPIPE project (White et al., 2011b), 0.04mm/s was applied for sweeps 1-4 and 0.15mm/s for sweeps 5 and 6. In the SAFEBUCK JIP project (White et al., 2011a), where testing was performed on a soft marine clay, 0.5mm/s was used but for sweep no. 22, 0.001mm/s was applied. In a direct shear test (White and Cathie, 2011), where normally consolidated kaolin clay was sheared on a steel interface, 0.003mm/s was applied. In the Monash Advanced Pipe-testing System (MAPS), the maximum velocity was 0.5mm/s and the minimum was 0.01mm/s. The selection of shearing rate in the current research program is intended to cover the undrained, partially drained and drained conditions in pipe axial walking.

No clear guidance is provided on how to transfer the pipe axial walking displacements into the shearing amplitude of direct simple shear tests. The ‘Standard Test Method for Consolidated Undrained Direct Simple Shear Testing of Cohesive Soils’ (ASTM-D6528, 2007) suggests at least a 20% shear strain for shear strength determination purpose.

2.6 Direct simple shear test

As discussed previously, cyclic direct simple shear testing was selected as the method for investigating the behaviour of soil within the shear zone. In this section, detailed knowledge of the direct simple shear test is presented in terms of its development, variety, uniformities in soil samples and the interpretation of test results.

2.6.1 Development of direct simple shear test

The direct simple shear test (DSS) is an improvement on the direct shear test, and both tests have been developed in order to investigate the shear strength of soil. The direct shear test

simulates the effect of shear loads acting on a predetermined failure surface. In these tests a circular or rectangular soil specimen is placed in a split box, as shown in Figure 2.12.

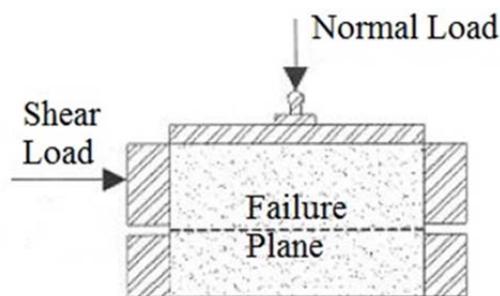


Figure 2.12 An illustration of the direct shear box (Zekkos et al., 2010)

In the first stage a normal load is applied at the top of the box and in the second stage, a shear load is applied at the top or bottom of the split box, which causes the specimen to shear along the predetermined failure plane between the two parts of the split box. During the test, the shear and normal stresses are measured and plotted against the horizontal displacement. A peak in the stress-strain curves is often observed. The progression of the normal displacement versus the horizontal displacement provides information about the contraction or dilation behaviour of the soil specimen (Grognet, 2011). In direct shear tests, the shear stress-strain distribution is highly non-uniform. This is due to a high concentration of stresses developing at the front and back edges of the soil specimen, which creates a progressive failure so that the total shear strength of the specimen is not fully mobilized. In addition, the strain restraints which force the failure in one direction create an unknown state of stress in the specimen (Terzaghi et al., 1996). The direct shear test is useful in determining the shear strength of soils, but not their stress-strain response (Bardet, 1997).

Due to the limitations of the direct shear tests, the direct simple shear test was introduced in order to achieve a more uniform specimen stress-strain distribution. Figure 2.13 illustrates the difference between the direct simple test and the direct shear test. In the direct simple test the soil specimen is confined by a wire-reinforced membrane or stacked rings. At the consolidation stage a normal stress is applied and the specimen is consolidated under one-dimensional conditions. The consolidation phase is followed by the shearing of the soil in one dimension only. This strain condition is called the simple shear strain condition (Dyvik et al., 1987).

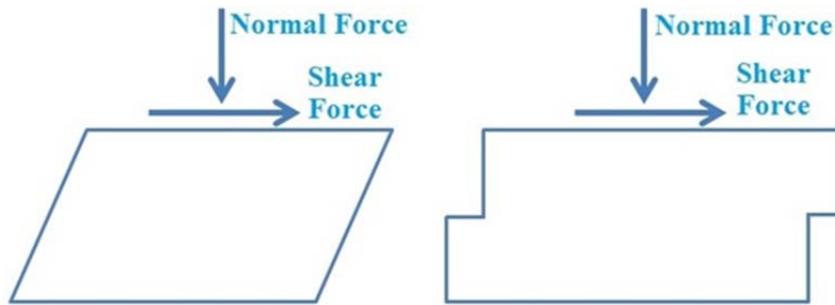


Figure 2.13 Illustrations of direct simple shear test (left) and direct shear test (right)

The first direct simple shear device was developed by the Swedish Geotechnical Institute (S.G.I) in 1936, as illustrated in Figure 2.14 below (Kjellman, 1951). This apparatus was able to test circular soil specimens with a height of 60mm and diameter of 20mm confined by a rubber membrane and aluminium rings. The sample was first consolidated using lead weights and drainage was allowed by placing porous stones at both the top and bottom of the soil specimen. The soil specimen was then sheared from the top plate, while maintaining either a constant load or constant sample height. To avoid slippage between the specimen and the plates, the top and bottom caps were often equipped with teeth. Vertical and horizontal load cells were used to record the normal stress and shear stress, and displacement gauges were applied to measure the vertical displacement and shear strain. The device was able to maintain a constant area of potential sliding during shearing, and to improve the homogeneity of the stress distribution when compared with the direct shear device.

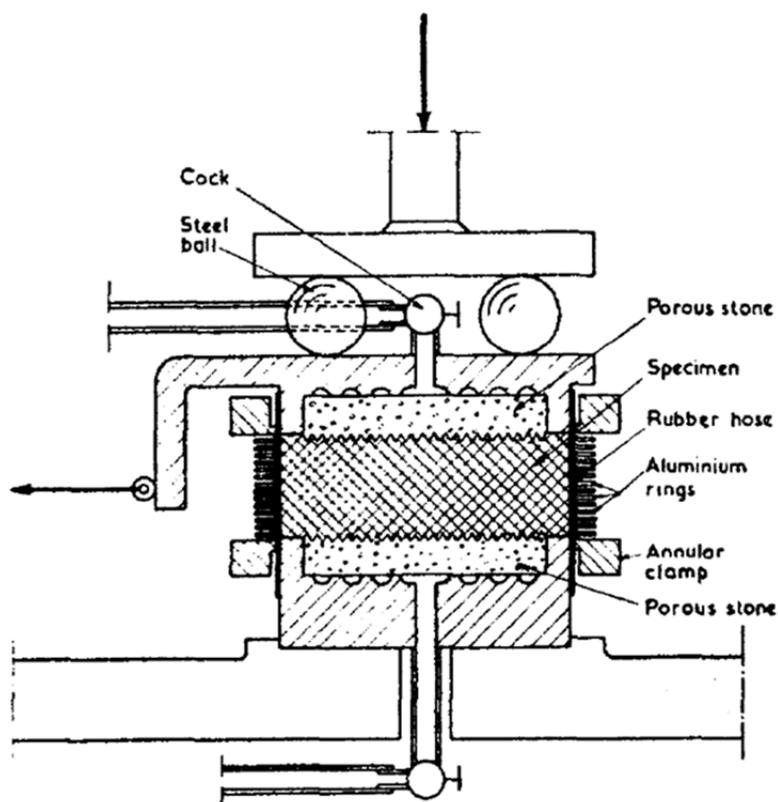


Figure 2.14 S.G.I Direct Simple Shear Device (Kjellman, 1951)

Kjellman's idea (Kjellman, 1951) of using a cylindrical sample confined with a reinforced rubber membrane was further developed by the Norwegian Geotechnical Institute (NGI) (Bjerrum and Landva, 1966), and a photograph of the apparatus is shown in Figure 2.15(a). A soil sample 80mm in diameter and 10mm high was confined in a rubber membrane reinforced with a spiral winding of wire with a diameter of 0.15mm wound at 25 turns per cm, as shown in Figure 2.15(b).

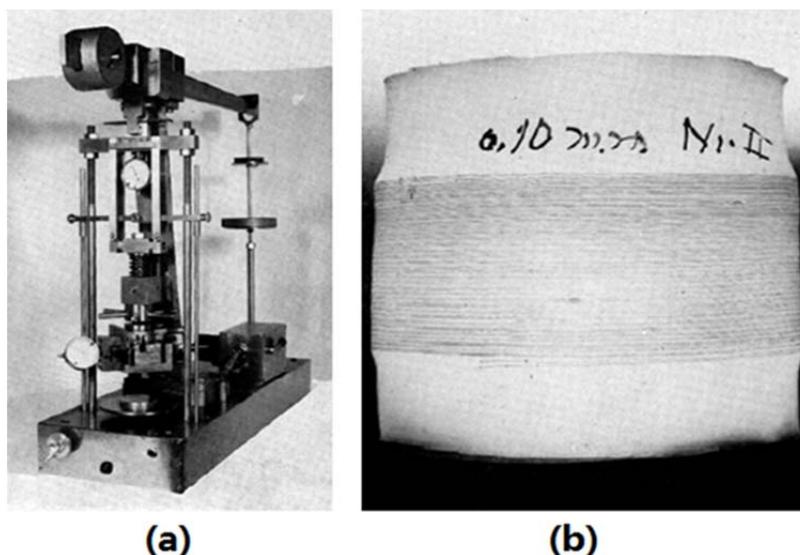


Figure 2.15 (a) NGI direct simple shear device (b) Reinforced rubber membrane (Bjerrum and Landva, 1966)

The confining membrane allows one-dimensional consolidation to occur while maintaining a constant cross-sectional area. At the consolidation stage, the major principal stress σ_1 is equal to the normal stress applied. During shearing a rotation of the principal stress will occur as a result of the increase in shear stress. The NGI direct simple shear device has since become the basis of the design of many later direct simple shear devices.

2.6.2 Drained and undrained direct simple shear test

Drained direct shear conditions are achieved by applying a constant normal load during shearing and applying an appropriate shearing rate so that no excess pore water pressure would build up during shearing. Drained direct simple shear testing is usually performed on granular materials such as sand, since the drained parameters of granular materials are most likely to be investigated.

For cohesive material like clay, the undrained parameters are most likely to be investigated. Therefore, for cohesive materials undrained direct simple shear testing is usually performed. However, several direct simple shear test devices are unable to perform truly undrained testing for two main reasons. Firstly, there may be difficulties in blocking the drainage in a direct simple shear test, because of the lack of pressurised cell which can provide back pressure. Secondly, pore pressure measurement may not be available when using direct simple shear devices.

2.6.3 Constant-volume undrained equivalent test

Since some apparatuses may not be able to perform truly undrained tests, undrained direct simple shear tests were carried out as constant-volume tests (Bjerrum, 1954). Bjerrum and Landva (1966) outlined that ‘during the shear phase of the constant-volume tests the specimens were drained and the rate of strain applied was selected such that the pore pressures in the specimen were zero throughout the tests; the height of the specimen was then kept constant by varying the vertical load on the sample’. Therefore, the undrained-equivalent constant-volume test is in essence a drained direct simple shear test with normal load variations.

Further, Bjerrum and Landva (1966) argued that in constant volume tests, the change in applied normal stress equals the change in pore water pressure which would have occurred in truly undrained direct simple shear tests. This argument was verified by Dyvik et al. (1987) for normally consolidated clay. Based on tests results, it was concluded that static undrained-equivalent constant volume tests on normally consolidated clays produce the same results as truly undrained tests. The assumption that the change in applied normal stress equals the change in pore water pressure which would have occurred in truly undrained direct simple shear tests was therefore verified. This is valid for saturated soils. The undrained-equivalent constant-volume direct simple shear test provides undrained parameters of cohesive soils. The American Society for Testing and Materials (ASTM) ‘Standard Test Method for Consolidated Undrained Direct Simple Shear Testing of Cohesive Soils’(ASTM-D6528, 2007) is based on the concept of constant volume testing, and many commercially-available direct simple shear devices are built according to ASTM-D6528, such as the GDS Shear base system used in the current research program.

2.6.4 The advantages of direct simple shear test over triaxial test

Airey and Wood (1987) argue that as a standard test the direct simple shear test has apparent merits when compared with the triaxial test. In direct simple shear testing it is relatively easy to set up the soil samples and consolidation takes place at a fast rate due to the small sample height. Furthermore, it is the only standard test that allows the rotations of principle stress axes. Such rotations take place in a large proportion of field situations and lead to a reduction in strength (Symes et al., 1984). However, there has been much discussion about the

applicability and suitability of the parameters obtained from direct simple shear testing (Saada and Townsend, 1981, Vucetic and Lacasse, 1982).

2.6.5 Non-uniformities in direct simple shear test

The direct simple shear test apparatus has been particularly criticised because it is unable to impose uniform stress on soil samples (Airey and Wood, 1987). The absence of complementary shear stress along the vertical boundaries of the soil sample means that ideal stress conditions cannot be achieved in the direct simple shear apparatus. As shown in Figure 2.16(a) in the test only the average normal stress, σ , the average shear stress, τ , and in some cases the average radial stress, σ_t are measured along the boundaries. Theoretical and experimental analyses have been conducted to investigate the relationship between the measured stresses and the stresses on a deformed soil sample in direct simple shear, as illustrated in Figure 2.16(b).

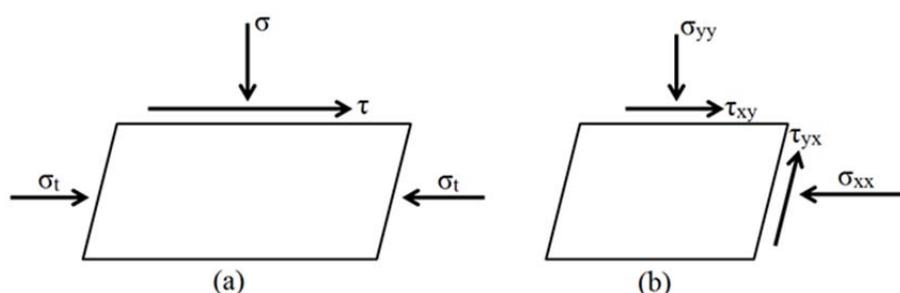


Figure 2.16 Stresses acting on test sample: (a) in direct simple shear apparatus; (b) in true simple shear (Airey and Wood, 1987)

Theoretical analyses have been performed using linear elastic samples (Lucks et al., 1972, Shen et al., 1978, Wright et al., 1978), but the results of these analyses give conflicting pictures of the degree of uniformity of stress. According to Lucks et al. (1972), over 70% of the sample was uniformly stressed, while Wright et al. (1978) concluded that direct simple shear tests cannot provide either reliable stress-strain relationships or absolute failure values. The test results from Vucetic and Lacasse (1982), who performed tests on real soils at various height-to-diameter ratios, showed that the measured soil behaviour is not affected by non-uniformities. In order to have a better understanding of stress non-uniformities, a circular simple shear device in which samples are surrounded by an array of load cells was developed (Budhu, 1979, Airey, 1984), so that the detailed stress-distribution can be measured. A comparison of the uniformity of stress in sands and clays was made by Airey and Wood

(1984) and they concluded that for the more plastic clay samples the uniformity improves significantly. Therefore, the results from direct simple shear tests on clay can be applied with more confidence than the results from tests on sands.

2.6.6 The effect of side boundaries on the uniformities of stress/strain

According to Roscoe (1953), under low normal stress conditions, the non-uniformities of strain are significant with straight vertical side boundaries, as shown in Figure 2.17 below. Despite the fact that the sides were made as smooth as possible, the sample did not deform as a parallelogram and separation could be observed at the corner with acute angles.

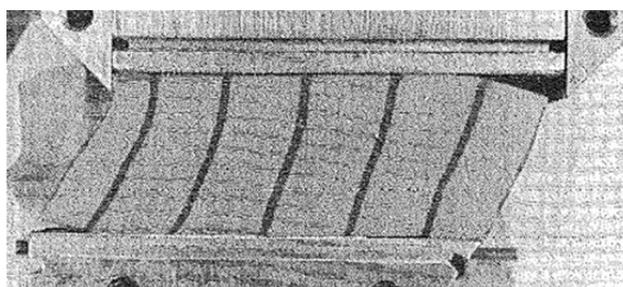


Figure 2.17 Side view of a plasticine sample after shearing under a normal stress of 12kPa (Roscoe, 1953)

Grognet (2011) developed an experimental device to investigate the effect of the vertical side boundaries on the uniformity of stress/strain in direct simple shear tests. Three different vertical side boundary configurations were considered, as shown in Figure 2.18 below.

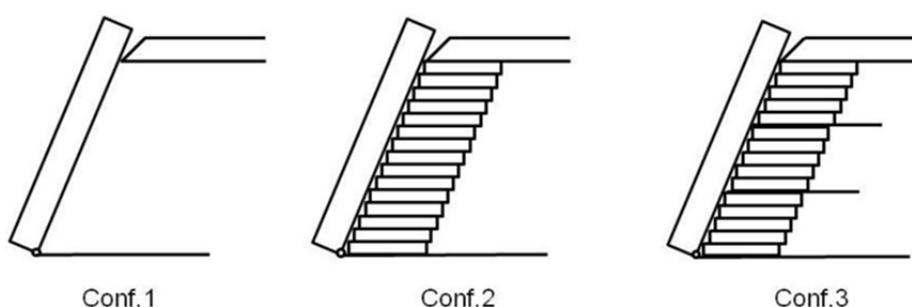


Figure 2.18 Three vertical boundary configurations considered (Grognet, 2011)

Configuration 1 is the conventional straight vertical side boundary, with two PVC plates enclosing the soil specimen. Configuration 2 is vertical boundaries comprising of a series of strips which can slide on top of each other, very similar to the stack of rings used in commercially-available Geonor devices. These strips are added to improve the stress/strain homogeneity by preventing tilting and separation. Configuration 3 is similar to Configuration

2, except that vanes intruding inside the soil specimen are provided, which can increase the number of corners along the vertical boundaries.

Grognon (2011) showed that with the presence of the strips and vanes, the stress/strain uniformity was improved compared with conventional straight vertical boundaries, and separation of the specimen at the acute corner is reduced. Based on the findings, the use of frictionless containing rings in the present commercially-available direct simple shear devices can improve the stress/strain uniformity.

2.7 Summary

A comprehensive literature review was conducted to build a conceptual framework upon which the research program was based. Offshore pipeline axial walking is the undesired accumulation of axial displacement of offshore pipelines during operation cycles. Research indicates that the axial resistance will tend towards the drained values during cycles of pipe movement, regardless of the rate or duration of each movement. A reliable prediction of this mechanism could bring significant design benefits, as drained resistance is usually higher than undrained resistance. Consequently, a higher range of axial resistance can be applied, leading to more cost-effective design. The pipe-soil interaction in pipe axial walking problems is extensively influenced by the soil response. The axial response is significantly influenced by the generation and dissipation of excess pore water pressure which leads to undrained and drained soil behaviour. It is likely that the response is between fully undrained and fully drained in typical field conditions. Analysis of the field data also suggests that the mobilisation of axial resistance is a time-related process instead of being linked to the distance of shearing. This suggestion is consistent with excess pore pressure development in cycles. In other words, the full pipe-soil resistance is only mobilised when steady pore pressures are reached. The residual resistance is the primary component of the load-displacement relationship in pipe axial walking, and it is dominated by the axial displacement rate.

The portion of the soil below the pipe which undergoes shearing, characterised as the shear zone, is significant for pipe axial walking assessment. Cyclic direct simple shear testing was selected as the preferred testing method to conduct an investigation into the soil behaviour within the shear zone owing to its various merits, including its loading pattern, allowing the rotation of principle stress axes, the small specimen size required, and its shear failure planes.

Chapter 3 - Experimental program and results

This chapter presents the experimental program and results. First the soil selected for the experimental program is introduced, followed by an elaboration of the experimental set-up. Finally the experimental results are presented.

3.1 Soil selection

The initial aim of the investigation was to simulate the environment of the petroleum pipelines which were to be laid on the seabed of the North Western Shelf, Australia. However, the use of actual soil material from the field was found to be not feasible due to both accessibility as well as quantity issues. As an alternative, as suggested by researchers at the University of Western Australia (UWA Centre for Offshore Foundation Systems), a kaolinite soil known as Prestige NY from Granville (NSW) was selected as the soil to be tested in the experimental program. Prestige NY kaolinite is commercially available and it was concluded that it exhibits similar characteristics to those of the seabed soil (Senthilkumar, 2013).

Various properties of both Prestige NY kaolinite and seabed silt have been investigated by Shannon (2013) and Senthilkumar (2013), including specific gravity analysis, particle size analysis, Atterberg limits tests, oedometer analysis, mineralogy analysis and triaxial testing.

3.1.1 General soil properties

The general soil properties of the Seabed silt and Prestige NY kaolinite clay are compared in Table 3.1, and the physical appearance of the soils is shown in Figure 3.1.

	Seabed silt	Prestige NY
Colour	Green	White
Swelling/Non-swelling	Non-swelling	Non-swelling



Figure 3.1 Soil sample of Prestige NY and Seabed Silt (Senthilkumar, 2013)

3.1.2 Specific Gravity

Specific gravity analysis was carried out for both soils, using an automatic density analyser. The measured specific gravity of both the Seabed silt and Prestige NY is given in Table 3.2 below.

Table 3.2 Specific Gravity of Seabed silt and Prestige NY (Senthilkumar, 2013)

Seabed silt	2.63
Prestige NY	2.61

3.1.3 Particle size analysis

The average particle size distribution of both soils is given in Figure 3.2. The distribution indicates that for both soils the majority of the particles are clay-sized.

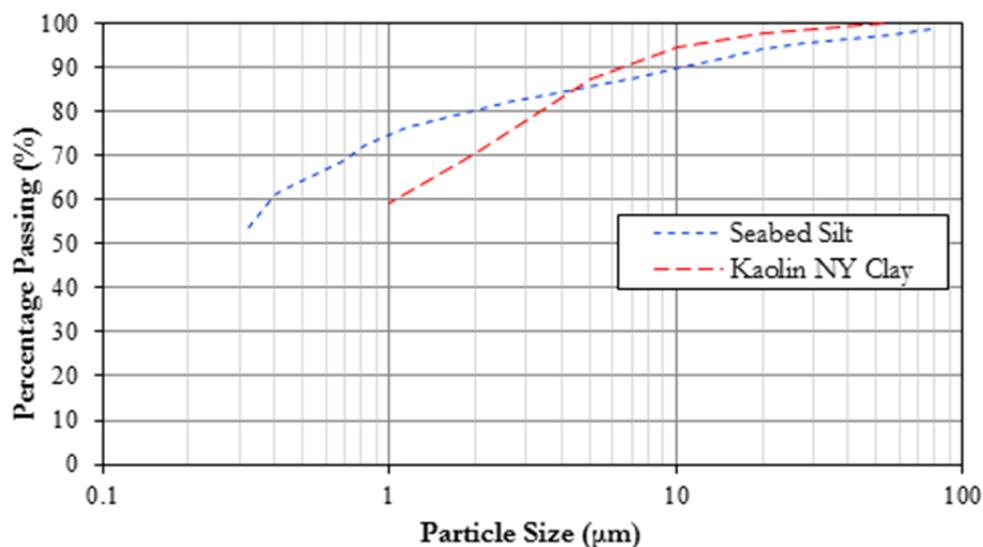


Figure 3.2 Particle size distribution curves (Senthilkumar, 2013, Shannon, 2013)

3.1.4 Atterberg limits analysis

The aims of the Atterberg limits analysis are to determine the liquid limit and plastic limit of the soil. The results are plotted on the USGS chart given in Figure 3.3. Both soils exhibit liquid limits as high as 50% of the gravimetric water content. According to the classification, the Prestige NY kaolinite is classified as inorganic highly plastic clay (CH) but falls close to the boundary of CL soils, whereas the seabed silt is classified as organic silt (MH). Despite the different classifications the two soils fall into, their close proximity on the USGS charts indicates that they exhibit similar wetting characteristics.

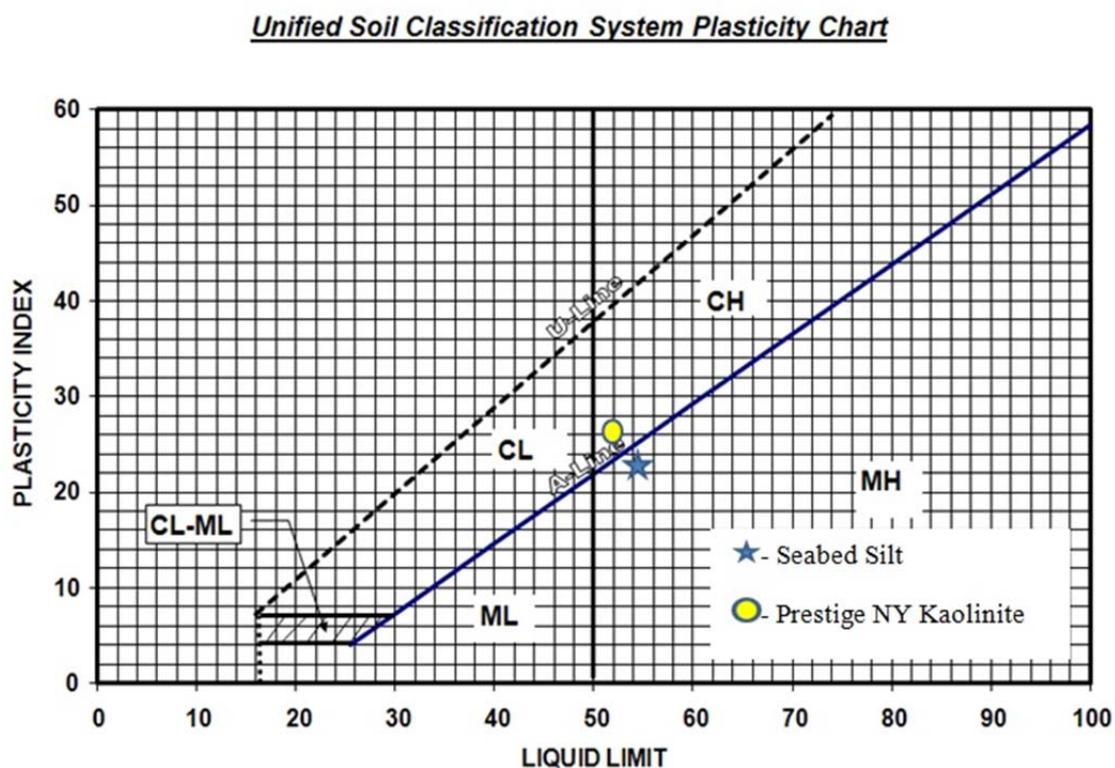


Figure 3.3 USGS Charts (Senthilkumar, 2013, Shannon, 2013)

3.1.5 Oedometer analysis

Oedometer tests were performed to determine the consolidation parameters of the two soils. The soils were prepared with a gravimetric water content of approximately twice the liquid limit and one-dimensional consolidation was implemented. The void ratio versus consolidation pressure curve is shown in Figure 3.4 and the consolidation parameters are given in Table 3.3.

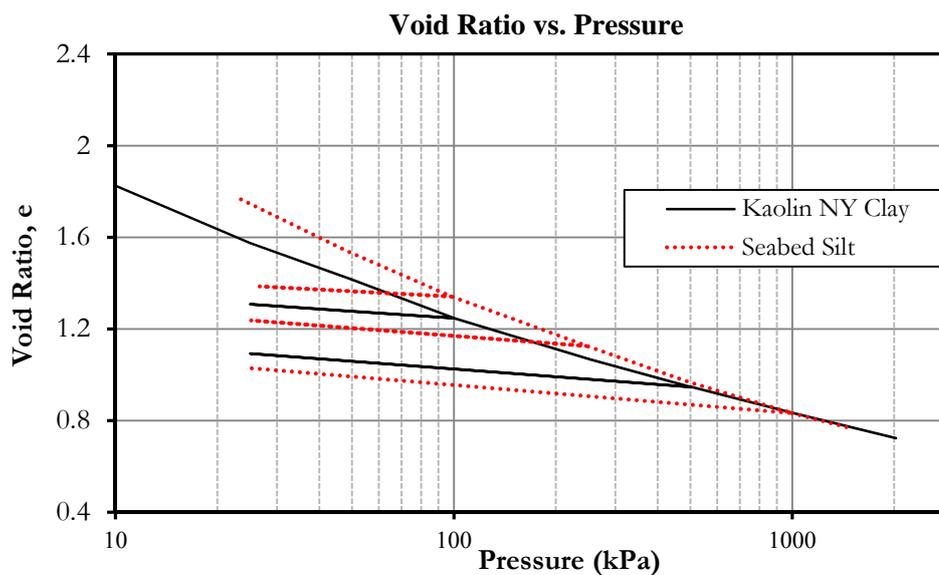


Figure 3.4 Consolidation and re-consolidation curves of oedometer tests (Senthilkumar, 2013, Shannon, 2013)

Table 3.3 Consolidation parameters from oedometer tests (Senthilkumar, 2013, Shannon, 2013)

	Seabed silt	Prestige NY
Initial Gravimetric Water Content (%)	120 $\approx 2 \times LL$	100 $\approx 2 \times LL$
Average Compression Index, c_c	0.46	0.40
Compressibility Parameter, λ	0.199	0.173
Average Swelling index, c_s	0.12	0.11
Unload/Reload Parameter, κ	0.053	0.046
Average Coefficient of Consolidation, c_v (m ² /sec)	1.2×10^{-7}	2.89×10^{-7}
Average Permeability (m/s)	1.8×10^{-9}	3.6×10^{-9}
Coefficient of Compressibility, m_v (kPa ⁻¹) at 50-100 kPa step	0.0016	0.0012
Secondary Compression, c_α	0.0058	0.0057

3.1.6 Triaxial tests

The main purpose of conducting the triaxial tests was to determine the undrained/drained friction angle and other relevant properties that can be applied in numerical modelling of soil behaviour. Table 3.4 below gives the parameters applied and the properties determined in the triaxial tests.

Table 3.4 Parameters and properties in triaxial tests (Senthilkumar, 2013, Shannon, 2013)

	Seabed silt	Prestige NY
Initial Gravimetric Water Content (%)	120 \approx 2 \times LL	100 \approx 2 \times LL
Drained/Undrained Friction Angle, ϕ' ($^{\circ}$)	19.65	21.9
Critical State Line, M	0.80	0.89
Average Coefficient of Consolidation, c_v (m^2/sec)	1.2×10^{-7}	2.19×10^{-7}

3.1.7 Summary

A summary of the average geotechnical properties of the seabed silt and the Prestige NY kaolinite is presented in Table 3.5. The closeness of the properties of both soils justifies the selection of Prestige NY kaolinite as the soil used in the experimental program.

Table 3.5 Geotechnical properties of Seabed silt and Prestige NY kaolinite (Senthilkumar, 2013, Shannon, 2013)

	Seabed silt	Prestige NY
Specific Gravity	2.63	2.61
Liquid Limit (%)	55	52
Average Compression Index, c_c	0.46	0.40
Compressibility Parameter, λ	0.199	0.173
Average Swelling Index, c_s	0.12	0.11
Unload/Reload Parameter, κ	0.053	0.046
Average Coefficient of Consolidation, c_v (m^2/sec)	1.2×10^{-7}	2.9×10^{-7}
Average Permeability (m/s)	1.8×10^{-9}	3.6×10^{-9}
Secondary Compression, c_α	0.0058	0.0057
Critical State Friction Angle, ϕ' ($^{\circ}$)	19.7	21.9
Critical State Line, M	0.8	0.85

3.2 Experimental set-up

3.2.1 GDS Direct Simple Shear Testing System

In the experimental program of the current research, cyclic direct simple shear tests were performed on Prestige NY kaolinite clay at low normal stress, using the GDS Standard Simple Shear System (GDS-STDSS), as shown in Figure 3.5. The GDS-STDSS system is an electro-mechanical shear testing device which is designed to meet and exceed the requirements of ASTM-D6528 (2007), ‘Standard Test Method for Consolidated Undrained Direct Simple Shear Testing of Cohesive Soils’.



Figure 3.5 GDS Standard Simple Shear System (GDS-STDSS)

The apparatus is a fully self-contained system with no requirements for compressed air or hanging weights. Normal (axial) and shear forces are applied using GDS electro-mechanical force actuators. Each axis (normal or shear) can be controlled in displacement (strain or velocity) mode as well as load or stress mode, which means that both constant-volume and constant-load tests can be performed. As the system is capable of performing cyclic shearing, it meets the needs of the research program. Axial and shear load readings are controlled under closed-loop feedback. Top cap fixity is assured through a system of crossed roller linear guides to minimise top cap rocking during shearing.

3.2.2 Specimen preparation

The Prestige NY kaolinite was first mixed from the powder state to a slurry using a soil mixer, achieving a 70% initial moisture content. The specimens tested in the GDS simple shear system are cylindrical specimens 50mm in diameter. The specimen is laterally confined by Teflon-coated low friction retaining rings, as shown in Figure 3.6(a), ensuring a constant cross-sectional area during shearing. The initial height of the specimen can be varied by adjusting the number of rings. During soil specimen preparation and insertion, the friction rings are first constrained by supporting forms and the reconstituted soil is inserted into the sample preparation apparatus, as shown in Figure 3.6(b). The assembly is then placed in the system as shown in Figure 3.6(c) and the supporting forms are removed. The top cap makes contact with the specimen and the hold-down assemblies are secured into place as shown in Figure 3.6(d).

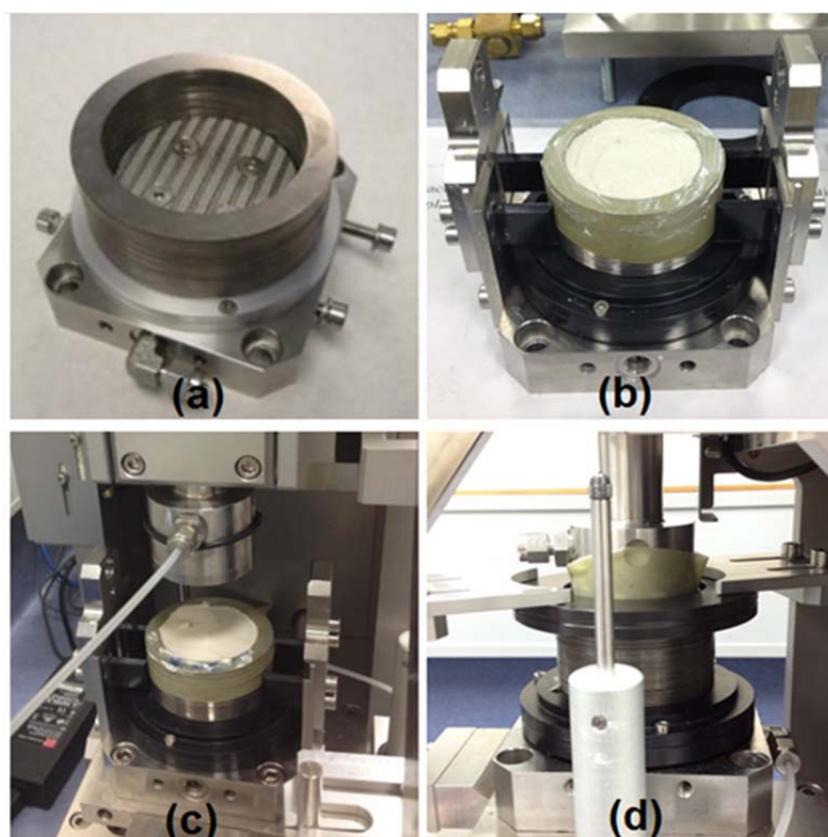


Figure 3.6 Specimen preparation and insertion

3.2.3 Pore pressure measurement

It is difficult to achieve truly undrained conditions in direct simple shear testing, unless a pressurised chamber is installed inside the system to host the sample. The GDS-STDSS

system was not originally designed to be capable of doing truly undrained tests and to be equipped with pore pressure measurement capacity. ASTM-D6528 (2007) specifies that constant-volume undrained-equivalent direct simple shear testing can be conducted to determine undrained soil properties in such systems. As presented previously, Bjerrum and Landva (1966) argued that in constant volume testing, the change in applied normal stress equals the change in pore water pressure which would have occurred in truly undrained direct simple shear testing. This argument was verified by Dyvik et al. (1987) for normally consolidated clay.

The intention of the current research program was not to establish truly undrained conditions in direct simple shear tests. The drainage was kept open at the consolidation stage as well as during cyclic shearing. However, when the applied shearing rate/velocity is not slow enough and consequently the soil response is not fully drained, some amount of excess pore pressure builds up, since dissipation of excess pore pressure is not fast enough. Therefore, pore pressure measurement capacity is still required to better interpret and analyse the test results.

Modification to the original GDS-STDSS system was carried out to equip the device with pore pressure measurement capacity. The base pedestal of the specimen set-up, where the bottom porous stone is attached, is connected to a pore pressure transducer (PPT) through the bottom drainage port as shown in Figure 3.7. When the base pedestal is saturated and the bottom drainage valve is closed, the reading from the PPT gives an estimation of the pore pressure level at the bottom of the specimen.

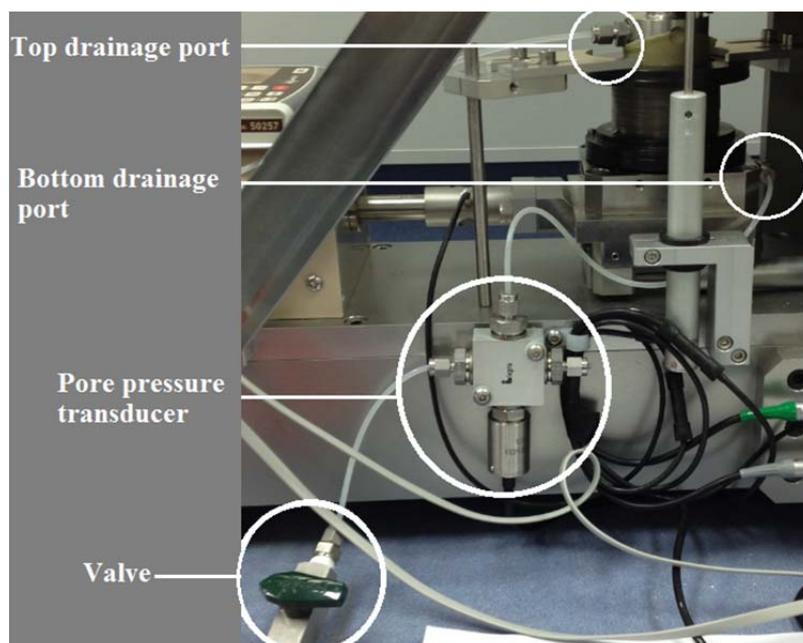


Figure 3.7 Pore pressure measurement configuration

The pore pressure transducer installed is the P620GDS-1MPa-C2.5 transducer produced by GDS Instruments. The transducer was connected to the GDS RS232 Data interface. Connecting the transducer to the data interface enables the transducer to work with the GDS-STDSS system under the same platform of GDSLAB, thus achieving simultaneous data recordings in the test.

Deionised water was used to saturate the transducer and the base pedestal. The water was first de-aired using a pump as illustrated in Figure 3.8(a). The base pedestal was immersed in water for 24 hours to make sure that it is fully saturated prior to sample insertion. The de-aired water was injected into the transducer through the bottom valve using a syringe, as shown in Figure 3.8(b). The readings of the PPT were closely monitored throughout the process to ensure that excess pore pressure was not built up.

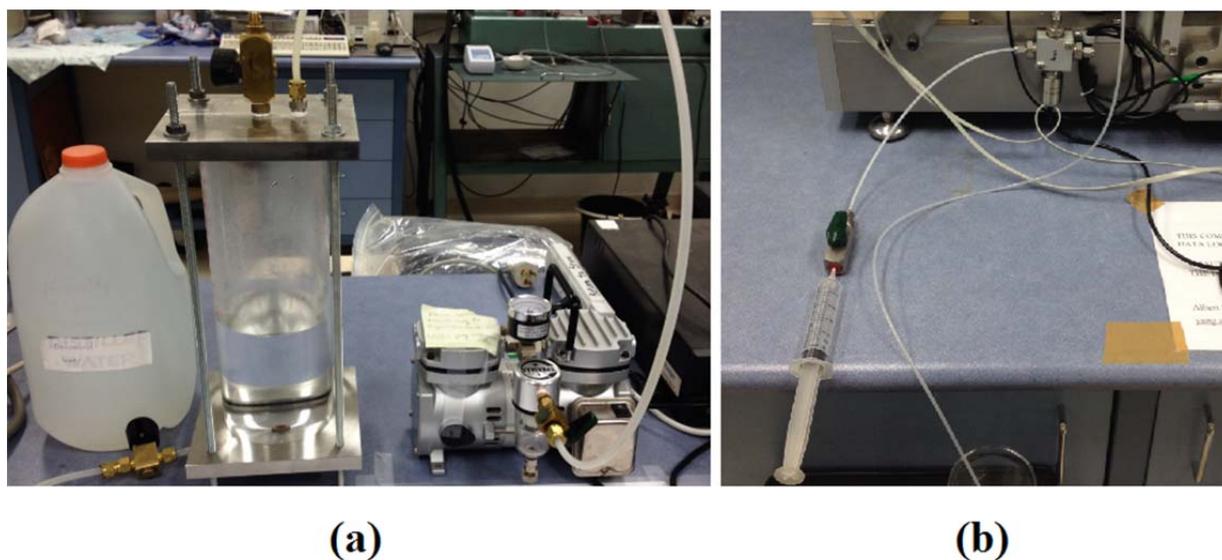


Figure 3.8 (a) De-airing water apparatus (b) Water injection

The bottom valve was closed when saturation was completed. The top drainage port was kept open during consolidation as well as the shearing stages, and the water coming out from the top drainage port was recorded using a measuring cylinder, as shown in Figure 3.9.

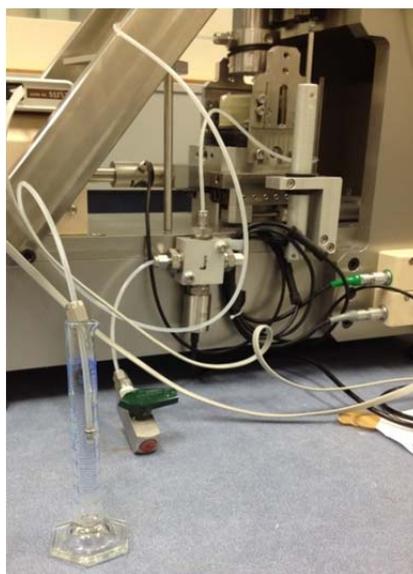


Figure 3.9 Measuring cylinder connecting to the top drainage port

3.3 Test methodology

3.3.1 Consolidation and normal stress

In the current experimental program, ideally the soft clay (Prestige NY kaolinite) would be normally consolidated one-dimensionally to an appropriate vertical/normal stress in the GDS-STDSS system. When the consolidation stage is completed, cyclic direct simple shear testing on the soil specimen would commence under the same normal stress.

As discussed previously, normally the magnitude of effective normal stress imposed on the soil underneath the pipe is relatively low in pipe axial walking problems. The effective normal stress (σ'_n) equals the total normal stress (σ_n) minus the pore water pressure (u). In drained cases, total normal stress (σ_n) may not be equal to the effective normal stress (σ'_n) since the pore water pressure can be very high due to the depth of the seabed, which can be in the order of kilometres. Based on the available literature (Bruton et al., 2007) , the effective normal stress imposed in pipe axial walking is in the range of 2kPa to 10kPa.

However, it was observed and confirmed in the tests that the friction from the horizontal motor rail of the GDS-STDSS system produced undesired variations in the horizontal/shear load readings, as demonstrated in Figure 3.10. In the tests presented in Figure 3.10, the horizontal motor moves back and forth in a sinusoidal function with an amplitude of 2mm, without a soil specimen in place. Vertical stresses of 0kPa, 20kPa and 40kPa were applied on top of the horizontal motor and the readings of the horizontal load cell were recorded.

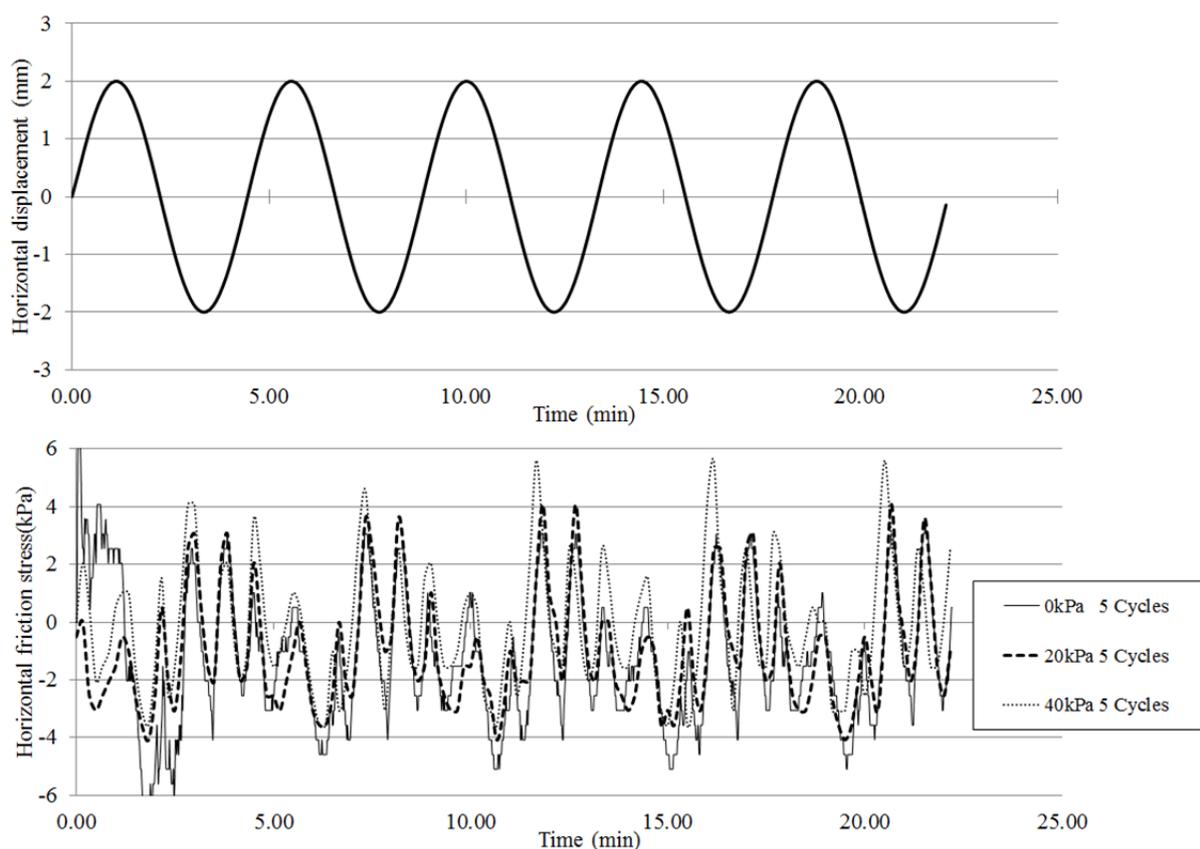


Figure 3.10 Variations in horizontal load readings due to rail friction

As Figure 3.10 shows, the magnitude of variations in horizontal/shear stress readings due to friction is between 2kPa and 5kPa, given the specimen diameter of 50mm. Several attempts were made to eliminate the variations in load readings due to rail friction, including correcting the raw horizontal load readings by offsetting the variations due to friction. However, due to the randomness of the variations, the outcome of the correction was deemed unsatisfactory. It is therefore concluded that the current testing system cannot produce satisfactory horizontal load readings under very low normal stress. The test results also show that the magnitude of the friction does not rise significantly with increasing applied vertical stress. As a result, a higher consolidation and normal stress of 60kPa was applied in the experimental program to minimise the effect of the rail friction.

3.3.2 Shearing velocities and shearing amplitude

The selection of shearing velocities in the current research program was intended to cover the undrained, partially drained and drained conditions in pipe axial walking.

Regarding the shearing amplitude, analysis of field data (White et al., 2011b) suggests that the mobilisation of axial resistance is a time-related process instead of being linked to the

distance of shearing. This suggestion is consistent with the excess pore pressure development in the sweeps. In other words, full pipe-soil resistance is only mobilised when steady pore pressures are reached. Combining this with the guidance from ASTM-D6528 (2007), a single shearing amplitude of 10% of specimen height was used in cyclic direct simple shearing. It is worth noting that the value of height in this case should be taken as the specimen height when the consolidation stage is completed.

3.3.3 Summary of test methodology

The Prestige NY kaolinite was first mixed from the powder state to a slurry using a soil mixer, achieving a 70% initial moisture content. The initial specimen height before consolidation was approximately 26mm. All soil specimens were first normally consolidated one-dimensionally in the GDS-STDSS system before shearing took place. Cyclic direct simple shear tests were performed on the normally consolidated specimens under a constant normal stress. The cyclic shearing followed a sinusoidal horizontal displacement function and the shearing amplitude was 10% of the specimen height. Various shearing velocities were applied, to cover the undrained, partially drained and fully drained responses.

3.4 Test results

3.4.1 Consolidation stage

The consolidation stages for all tests reported here were performed identically. The Prestige NY kaolinite was first mixed from the powder state to a slurry using a soil mixer, and staged consolidation was carried out within the GDS STDSS system. First, a consolidation stress of 20kPa was applied and the consolidation stress was increased to 40kPa and 60kPa subsequently. Table 3.6 outlines the key soil properties and test parameters at the consolidation stage.

Table 3.6 Soil properties and test parameters at consolidation stage

Average initial soil moisture content	71.13%	
Average initial soil wet density	1.42g/cm ³	
Initial soil specimen height	26.36mm	
	Consolidation Stress (kPa)	Consolidation Time (min)
	20kPa	120
Staged Consolidation	40kPa	120
	60kPa	until pore water pressure level stabilised

The consolidation displacement vs. time curves are presented in Figure 3.11 . Figure 3.11(a) is the plot of consolidation displacement in mm versus elapsed time in minutes. Figure 3.11(b) is the plot of consolidation displacement as a percentage of final displacement versus normalised time $c_v t/H^2$, where

c_v is the average coefficient of consolidation, taken as $2.9 \times 10^{-7} \text{m}^2/\text{sec}$;

t is the elapsed time in seconds;

H is the initial soil specimen height in meters, taken as 0.02636m. The bottom drainage port was closed during consolidation as one-way drainage is considered.

As demonstrated in Figure 3.11, due to the relatively small specimen size in the GDS-STDSS system, the consolidation stage can be completed within a normalised time of $c_v t/H^2 = 12$ (12 hours).

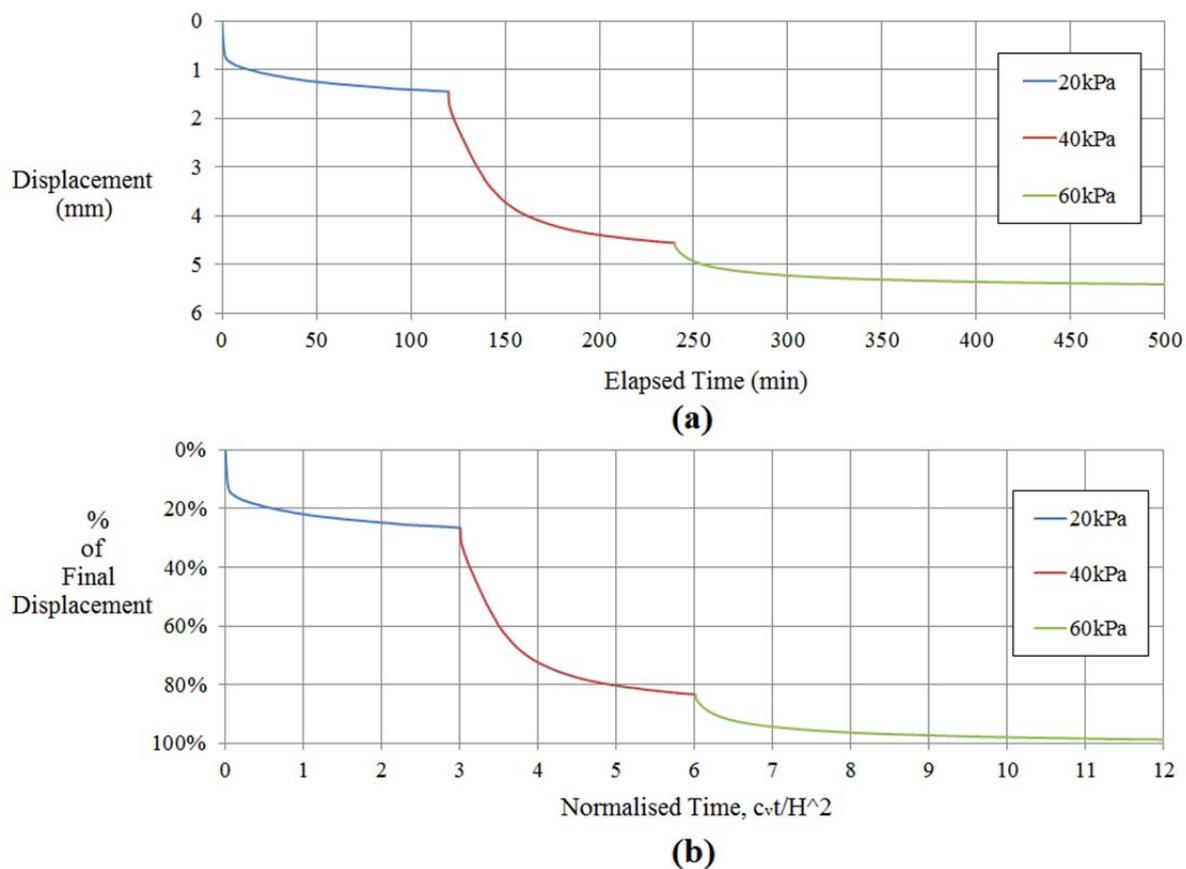


Figure 3.11 Consolidation displacement vs. (a) Elapsed time (b) Normalised time

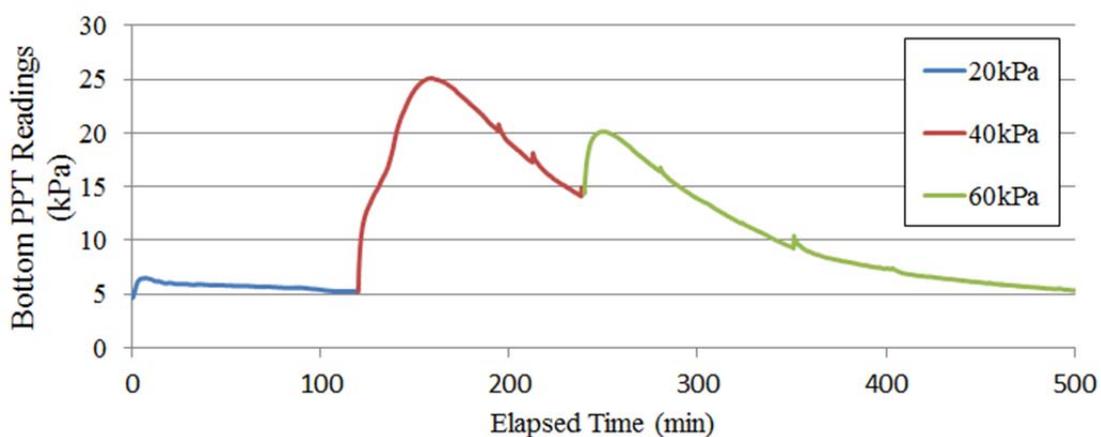


Figure 3.12 PPT Readings at consolidation stage

The readings of the pore pressure transducer (PPT) at the consolidation stage are plotted in Figure 3.12. The readings show that there is a delayed response from the PPT readings. To be more specific, when the consolidation stress increases, the PPT readings do not rise immediately to a peak. Instead, a gradual increase or delayed response is observed from the readings. Further, the change in PPT reading is not equal to the actual excess pore pressure

that builds up in the specimen. As introduced earlier, due to the limitations of the experimental set-up, the PPT was connected to the bottom drainage port instead of within the specimen. Therefore, the PPT can only provide an estimation of the pore pressure level at the bottom drainage port. Nevertheless, the PPT readings can provide some useful data when analysing the excess pore pressure development in the specimen at both consolidation and cyclic direct simple shearing stages.

3.4.2 Cyclic shearing stage

Cyclic direct simple shear tests were performed on the normally consolidated specimens under a constant normal stress applied. Table 3.7 summarises the test parameters at the cyclic shearing stage.

Table 3.7 Test parameters at the cyclic shearing stage

Applied normal stress during shearing	Average specimen height before shearing	Shearing amplitude	Shearing velocities	No. of cycles
60kPa	19.50mm	10% of specimen height	0.3mm/s 0.03mm/s 0.005mm/s 0.0005mm/s	10-30

The application of a constant normal stress is consistent with the vertical loading condition of the soil underneath the pipe in pipe axial walking, where the vertical load is due to the pipe's weight. Similar to the drainage path of the soil underneath the pipe where there is free drainage at the soil surface, the top drainage port was kept open at the shearing stage. The shearing amplitude was 10% of the specimen height. Various shearing velocities were applied, to cover the undrained, partially drained and fully drained responses. The period of one cycle ranges from approximately 26 seconds for fast shearing to 1560 seconds for slow shearing.

The specimen was sheared by mobilising the horizontal load motor, as shown in Figure 3.13, and the horizontal displacement followed a sinusoidal function.

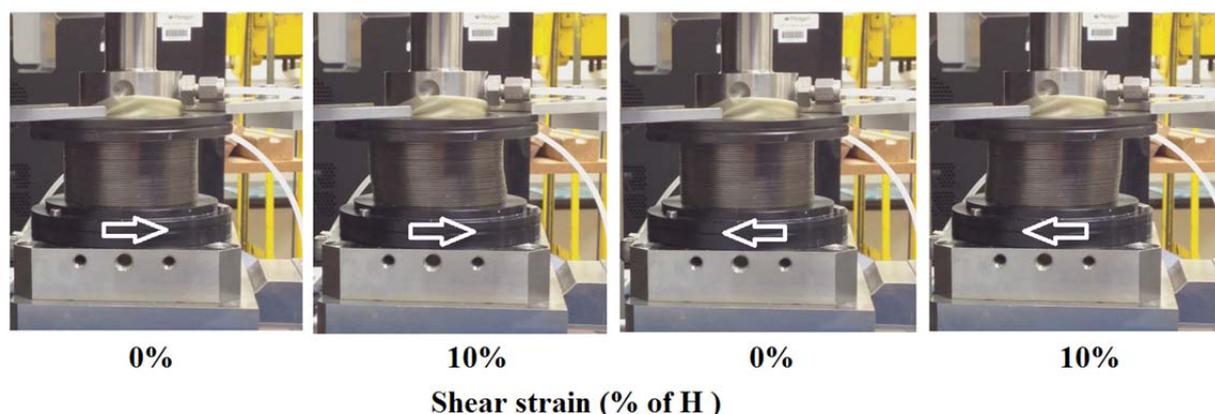


Figure 3.13 Cyclic shearing of the soil specimen

Pore pressure development

Shearing velocities significantly affect the level of the excess pore pressure that builds up during cyclic shearing, and therefore dominate whether the soil response in the shear zone is undrained, partially drained or fully drained in pipe axial walking. The readings of the pore pressure transducer were recorded and Figure 3.14 presents the PPT readings versus time plots. The shearing velocities, from fastest to slowest, are (a) 0.3mm/s, (b) 0.03mm/s, (c) 0.005mm/s and (d) 0.0005mm/s.

The shear strain versus time curves are also plotted on the same figure for each shearing velocity. The plots show that for high shearing velocities like those shown in Figures 3.14(a) & (b), excess pore pressure continuously builds up as cyclic shearing progresses, and higher velocity induces higher levels of excess pore pressure. The fast shearing cases can be characterised as undrained shearing. If the shearing velocity is moderate, like that shown in Figure 3.14(c), excess pore pressure is generated and plateaus after reaching a moderate level. As cyclic shearing continues, excess pore pressure dissipates eventually. The moderate shearing cases can be characterised as partially drained. For very slow shearing like that shown in Figure 3.14(d), the level of the excess pore pressure that builds up during shearing is negligible compared with faster cases. Therefore, the slow shearing cases can be characterised as drained.

Continuous consolidation

Continuous consolidation of the specimens was observed as the cyclic shearing progressed. This is due to the fact that free drainage at the top of the soil is permitted during cyclic shearing. This drainage path is similar to the drainage path of the soil in the shear zone underneath the axially-walking pipe, where water is free to drain from the soil surface. Figure

3.15 illustrates the vertical displacement (as a percentage of specimen height H) versus shear strain plots at the cyclic shearing stage. The shearing velocities, from fastest to slowest, are (a) 0.3mm/s, (b) 0.03mm/s, (c) 0.005mm/s and (d) 0.0005mm/s.

When the shearing velocity is 0.3mm/s, the specimen consolidates continuously without a clear tendency towards convergence. However, the increase of vertical displacement is relatively small, only passing 5% of specimen height H after 30 cycles. Under a shearing velocity of 0.03mm/s, the vertical displacement increases to 5% of H after 5 cycles and there is a clear tendency towards convergence. The vertical displacement converges to 12% of H after 20 cycles. For the slow shearing cases of 0.005mm/s and 0.0005mm/s, the vertical displacement increases to 5% of H after 2 cycles and 1 cycle respectively. There are clear tendencies towards convergence and the vertical displacements converge to 13% of H for 0.005mm/s and 17% of H for 0.0005mm/s.

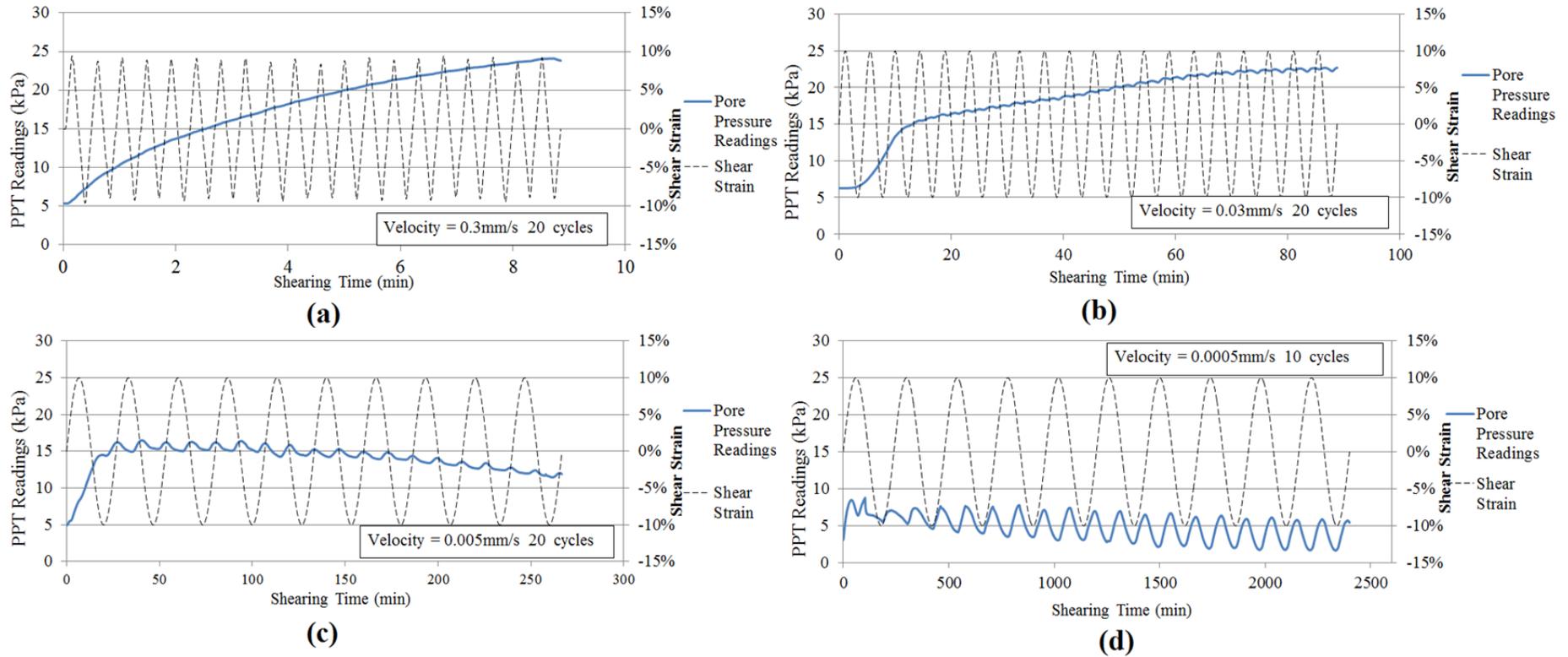


Figure 3.14 Excess pore pressure development at shearing stage

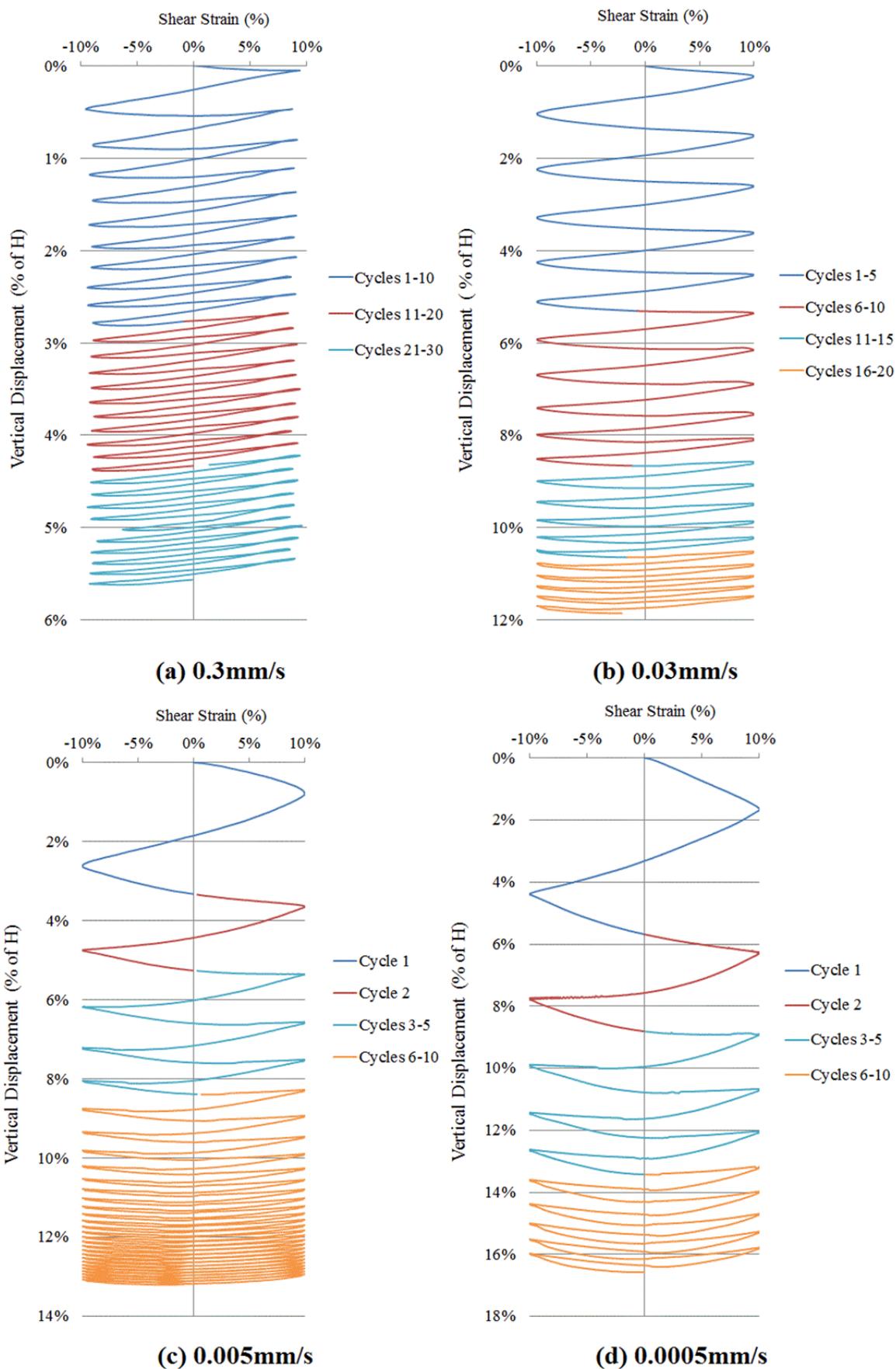


Figure 3.15 Continuous consolidation at shearing stage

Shear stress-strain relationship

The shear stress-strain relationships at the cyclic shearing stage are plotted on Figure 3.16 for shearing velocities (a) 0.3mm/s;(b) 0.03mm/s and Figure 3.17 for shearing velocities (a) 0.005mm/s; (b) 0.0005mm/s. The level of shear resistance is presented as $\tau/\sigma_{\text{total}}$, where τ is the shear/horizontal stress and σ_{total} is the total normal stress applied.

At a high shearing velocity of 0.3mm/s as shown in Figure 3.16(a), a peak or breakout shear resistance is observed when the specimen is sheared for the first time, reaching a peak of just over $\tau/\sigma_{\text{total}}=0.4$. The shear resistance gradually reduces to a residual level of $\tau/\sigma_{\text{total}}=0.17$ after 5 cycles and the shear stress-strain relationship remains stable for the subsequent cycles, representing a ductile response.

At a moderate shearing velocity of 0.03mm/s as shown in Figure 3.16(b), a peak shear resistance takes place when the specimen is first loaded, reaching a peak of approximately $\tau/\sigma_{\text{total}}=0.25$. The shear resistance reduces to a residual level and shows ductile response in cycle 2. The response stays ductile for subsequent cycles. However, the level of residual shear resistance increases gradually as cyclic shearing continues. This is due to the fact that excess pore pressure dissipates and effective stress rises as shearing continues, resulting in higher shear resistance. The residual shear resistance is close to $\tau/\sigma_{\text{total}}=0.2$ in cycles 2-5 but gradually grows to $\tau/\sigma_{\text{total}}=0.32$ in cycles 6-20.

At low shearing velocities of 0.005mm/s and 0.0005mm/s as shown in Figure 3.17 (a) and (b) respectively, only ductile response is observed when the specimen is first sheared, and the shear resistance is always residual. The level of residual shear resistance increases gradually as excess pore pressure dissipates and effective stress rises, and the shear resistance converges to $\tau/\sigma_{\text{total}}=0.46$ for 0.005mm/s and to $\tau/\sigma_{\text{total}}=0.52$ for 0.0005mm/s.

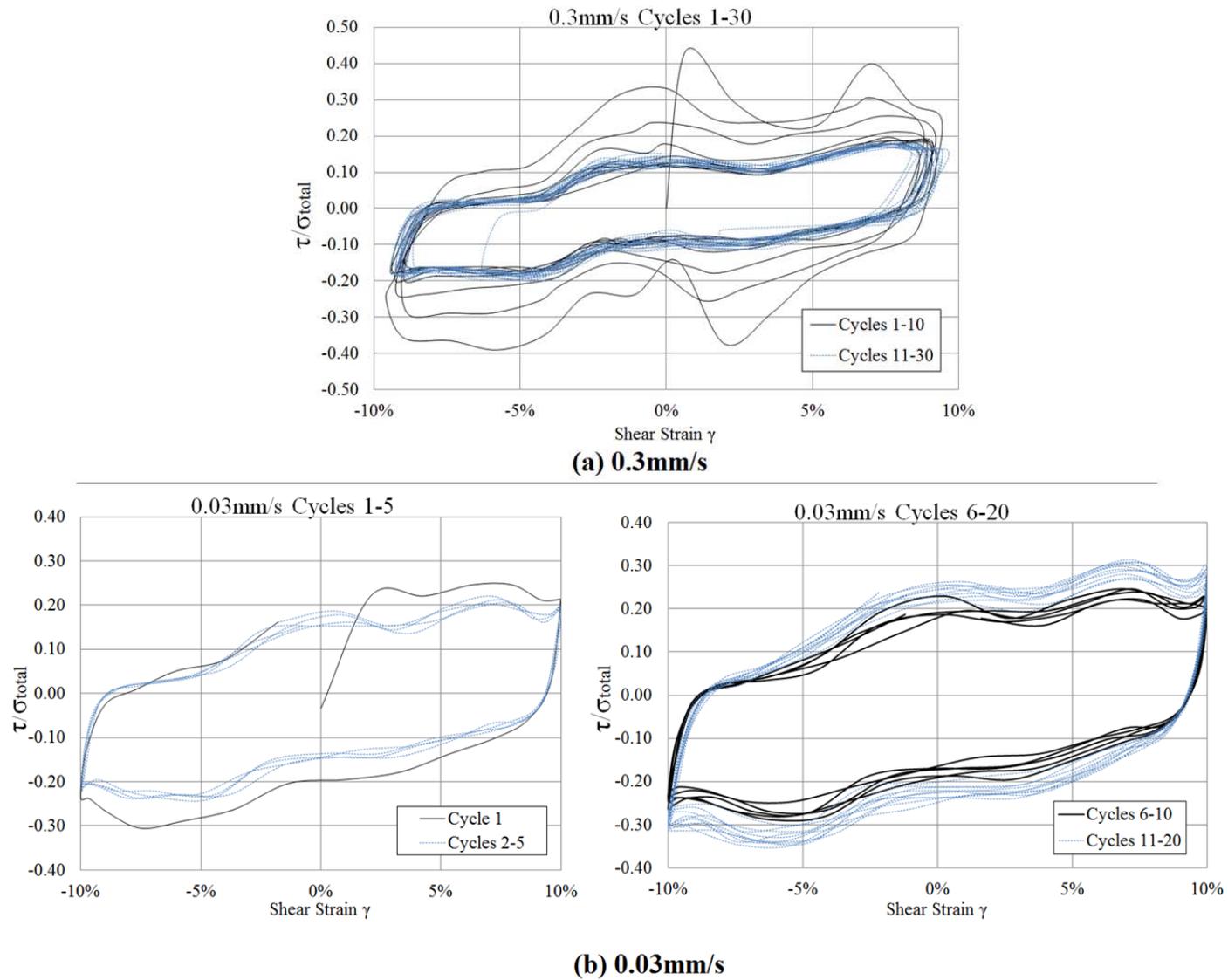


Figure 3.16 Shear stress-strain relationship for (a) 0.3mm/s and (b) 0.03mm/s

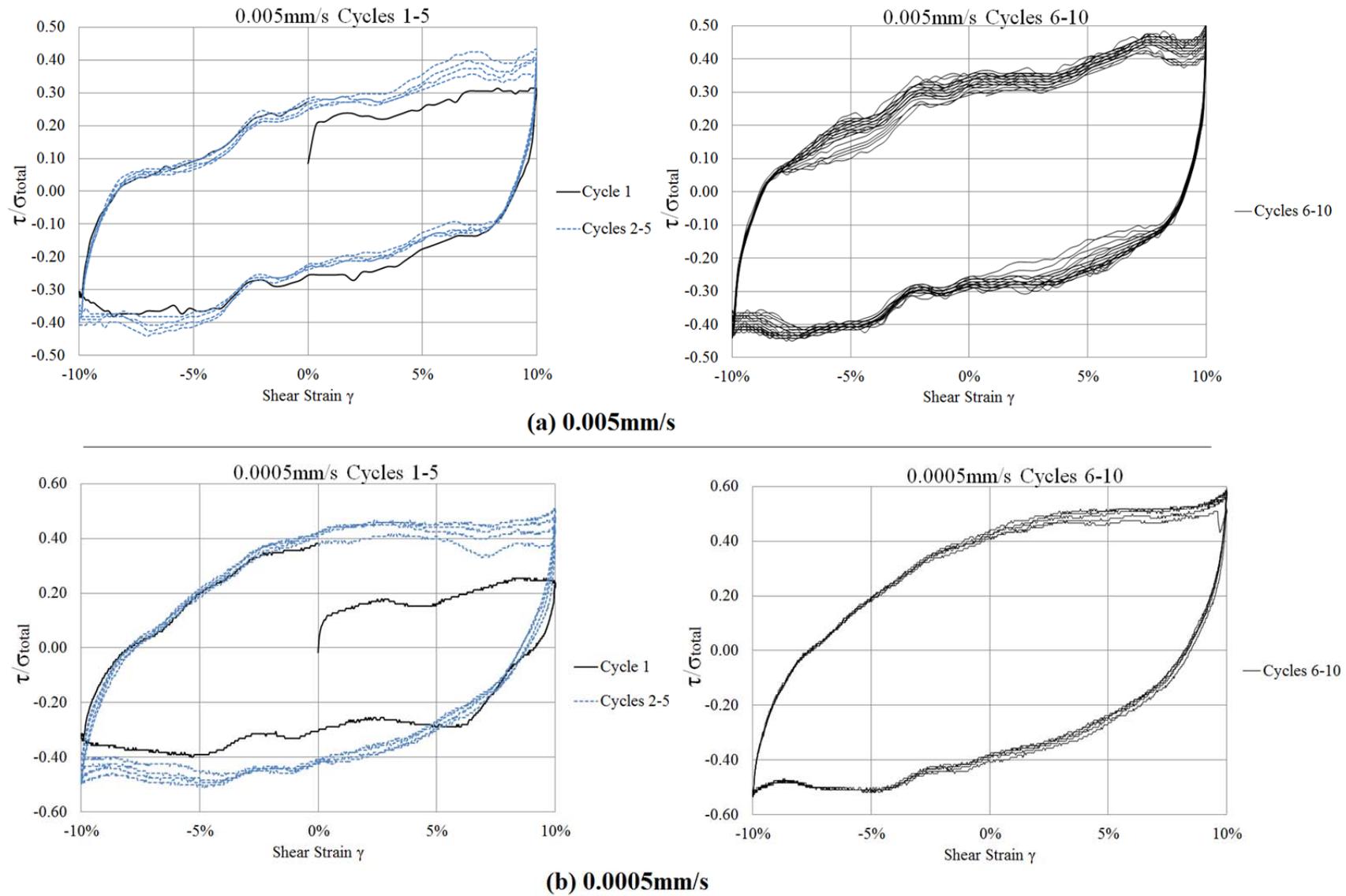


Figure 3.17 Shear stress-strain relationship for (a) 0.005mm/s and (b) 0.0005mm/s

3.5 Concluding remarks

Shearing velocities significantly affect the level of the excess pore pressure that builds up during cyclic shearing, and therefore dominate whether the soil response in the shear zone is undrained, partially drained or fully drained in pipe axial walking. At a high shearing velocity, excess pore pressure builds up continuously and the shearing is undrained. At a moderate shearing velocity, excess pore pressure builds up and dissipates and effective stress rises eventually, resulting in higher shear resistance. At a low shearing velocity, no excess pore pressure builds up and the shearing is drained.

Continuous consolidation of the specimens was observed as the cyclic shearing progressed. This is due to the fact that free drainage at the top of the soil is permitted during cyclic shearing. This drainage path is similar to the drainage path of the soil in the shear zone underneath the axially-walking pipe, where water is free to drain from the soil surface.

The residual shear resistance in cyclic shearing indicates the level of axial soil resistance in the shear zone when offshore pipeline axial walking occurs. Figure 3.18 shows the residual shear resistance versus velocity plot for the cyclic direct simple shear tests performed. The level of shear resistance is presented as τ/σ_{total} , where τ is the shear/horizontal stress and σ_{total} is the total normal stress applied. The shearing velocity is normalised as vH/c_v , where

v is the shearing velocity in m/sec;

H is the soil specimen height before shearing, taken as 0.0195m.

c_v is the average coefficient of consolidation, taken as $2.9 \times 10^{-7} \text{ m}^2/\text{sec}$;

The residual resistance levels versus axial displacement rate plots in MAPS tests are also presented in Figure 3.18(Senthilkumar, 2013). The results are from the tests with smooth pipe surface at various pipe embedment depths. The axial pipe residual resistance is normalised by the maximum recorded vertical load V_{max} and the axial displacement rate is normalised by the effective pipe diameter D' and the coefficient of consolidation C_v .

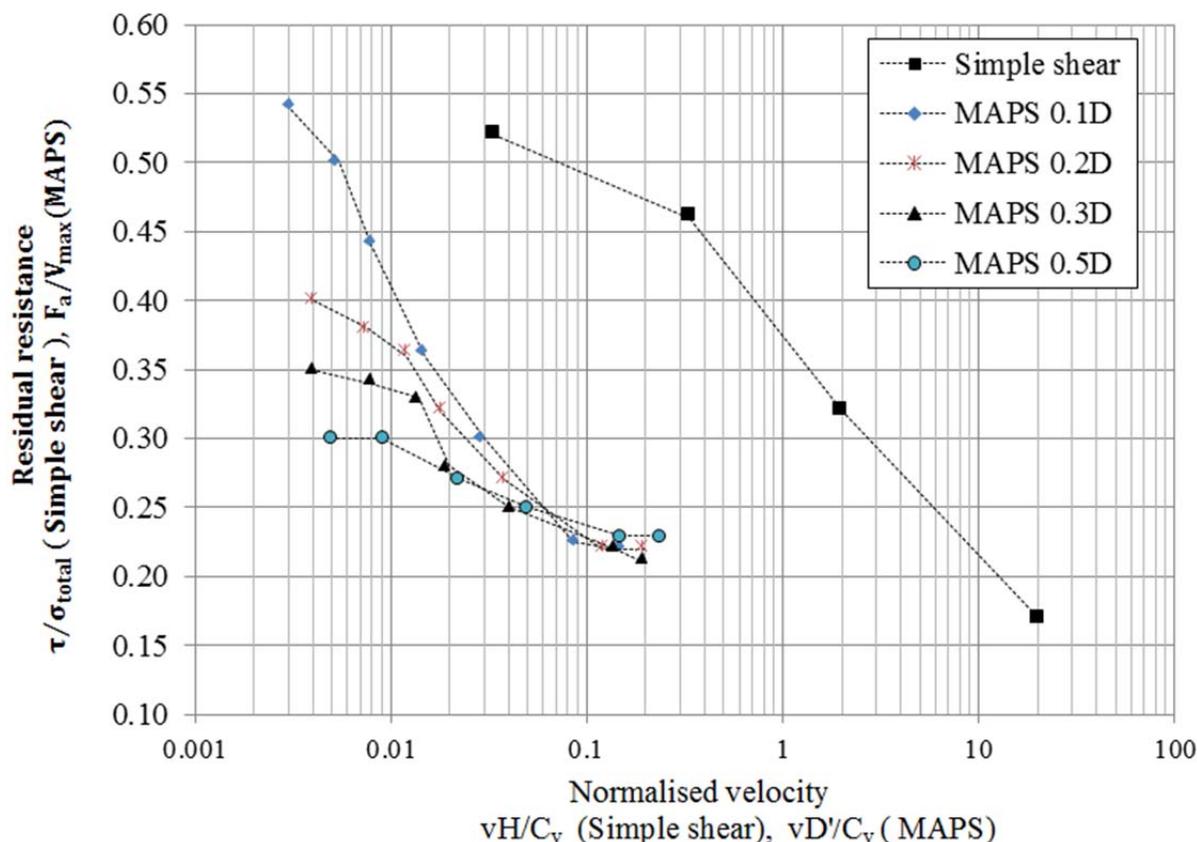


Figure 3.18 Residual resistance versus normalised velocity

The residual resistance in cyclic direct simple shear tests indicates the level of shear resistance of the soil in the shear zone, while the residual resistance in MAPS tests represents the pipe-soil interface resistance in the axial direction. The velocity in the direct simple shear tests is normalised by the specimen height, whereas the velocity in the MAPS tests is normalised by the effective pipe diameter. The above facts mean that the relationships between residual shear resistance and shearing velocity in direct simple shear tests and in MAPS tests do not match exactly with each other. Nevertheless, Figure 3.18 suggests that some shifting factors or transferring equations may be developed to relate the residual resistance as well as the undrained/drained limit in direct simple shear tests to those in pipe-soil interface interaction in the axial direction. If standard test methods such as the current direct simple shear test can be applied in practical design taking offshore pipeline axial walking into consideration, the process would be significantly simplified in terms of time and cost.

More results for cyclic direct simple shear tests can be obtained from finite element analysis and a set of data can be established which can be applied in the calibration of large-scale pipe

axial walking modelling, and provide guidance on the design practice of offshore pipelines when considering on-bottom stability in axial direction.

Chapter 4 – Numerical Modelling

The numerical modelling work of the current research program is presented in this chapter. Finite element analyses were conducted to capture the behaviour of soil within the shear zone, using the Abaqus Standard finite element analysis software package. Abaqus is a suite of powerful engineering simulation programs based on the finite element method and it is capable of solving a wide range of problems from simple linear analysis to complicated nonlinear behaviour. In particular, for this research program, the soil constitutive model is capable of incorporating shear-induced pore water pressure generation. This requires an advanced constitutive soil model such as Modified Cam-clay model, which is included in the Abaqus standard package.

The critical state soil theory, upon which the Modified Cam-clay model is based, and the model itself are introduced in this chapter, followed by the two-dimensional finite element analysis (FEA) of cyclic direct simple shearing on soft clay, and the results of the analyses are presented.

4.1 Finite element model

4.1.1 Critical state soil theory and modified Cam-clay model

The specific model implemented in Abaqus Standard is an extension of the Modified Cam-clay model (Abaqus 6.13 Online Documentation). The model is based on the critical state soil theory developed by researchers at Cambridge University (Roscoe et al., 1958, Burland and Roscoe, 1969, Schofield and Wroth, 1968). The model is capable of describing the stress-strain behaviour of soft soils. In particular, the model can predict pressure-dependent soil strength and the compression and dilatancy (volume change) caused by shearing. Because the model is based on critical state theory, it predicts unlimited soil deformations without changes in stress or volume when the critical state is reached.

Detailed elaborations of the critical state soil theory and the modified Cam-clay model are provided in Appendix A.

4.1.2 Two-dimensional finite element model

A two-dimensional finite element model has been developed in Abaqus to simulate cyclic direct simple shearing on soft clay (Prestige NY kaolinite). The dimensions of the 2-D model are consistent with the dimensions of the soil specimen in the experimental program, as shown in Figure 4.1. The initial height of the model (H) is 18.67mm and the length of the model (L) is 50mm. The soil specimen in the experimental program is a cylinder of 50mm diameter and an average initial height of 26.36mm. The soil is represented by 4-node bilinear displacement and pore pressure element (CPE4P).

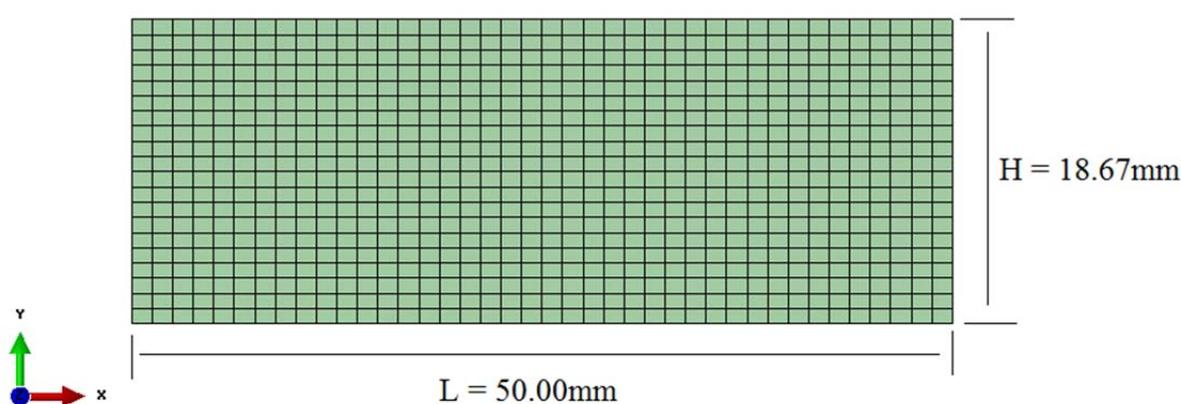


Figure 4.1 Two-dimensional finite element model

4.1.3 Material properties

Table 4.1 presents the soil properties in the modified Cam-clay model that was applied to simulate the elastic and plastic behaviour of the soft clay (Prestige NY kaolinite) in finite element analysis. The soil properties of the Prestige NY kaolinite are based on the various tests conducted previously at Monash University, Australia (Shannon, 2013, Senthilkumar, 2013) and the initial soil conditions are consistent with the initial soil conditions in the experimental program. It was assumed that the soil is fully saturated (i.e. $S=1$) and the initial water content (ω_0) was equal to 70%.

Table 4.1 Material inputs for the finite element modelling

Initial conditions		
Specific gravity, G_s		2.61
Degree of saturation, S		1
Initial water content, ω_0		70%
Initial void ratio, e_0		1.827
Initial saturate density, ρ_{sat}		1.57g/cm ³
Modified Cam-clay inputs		
	Shear = Poisson	
Porous Elastic	Logarithmic elastic bulk modulus, κ	0.040
	Poisson's ratio, ν	0.3
Clay Plasticity	Intercept, N	2.10
	Logarithmic plastic bulk modulus, λ	0.174
	Stress ratio at critical state, M	0.85

In the 'Porous Elastic' Modified Cam-clay inputs, the 'Shear=Poisson' option was chosen to compute the instantaneous shear modulus G from the bulk modulus and Poisson's ratio. The Poisson's ratio was assumed to be a constant of 0.3 under drained conditions.

To properly simulate the pore pressure development in the soil, the vertical permeability at different void ratio levels was defined as given in Table 4.2. The permeability input was based on the experimental investigation of the vertical and horizontal permeability of kaolin clay by Al-Tabbaa and Wood (1987) as well as on the tests performed at Monash University, Australia (Shannon, 2013, Senthilkumar, 2013)

Table 4.2 Vertical permeability for different void ratio levels

Void ratio, e	Vertical permeability, k_v , in m/s
0.8	2.6184E-10
0.9	3.7991E-10
1.0	5.3000E-10
1.1	7.1627E-10
1.2	9.4295E-10
1.3	1.2143E-09
1.4	1.5348E-09
1.5	1.9086E-09
1.6	2.3404E-09
1.7	2.8346E-09
1.8	3.3958E-09
1.9	4.0284E-09
2.0	4.7373E-09
2.1	5.5270E-09
2.2	6.4022E-09
2.3	7.3678E-09
2.4	8.4284E-09
2.5	9.5888E-09

4.1.4 Load and boundary conditions

Consolidation stage

At the consolidation stage, the soil is normally consolidated and a constant consolidation stress is applied uniformly on the top surface of the soil model. Pore water is free to drain from the top surface and drainage is not allowed at the bottom, as in the experimental set-up. The model is constrained on both the left and right vertical sides and no displacement is allowed in the horizontal direction (x-direction). The model can be compressed one-dimensionally in the vertical direction (y-direction) at the consolidation stage.

Cyclic shearing stage

At the cyclic shearing stage, the soil is subjected to a constant vertical stress applied uniformly on the top surface of the soil model. Pore water is free to drain from the top surface and drainage is not allowed at the bottom. The constraints on the vertical sides are released and displacement is allowed in both the horizontal direction (x-direction) and the vertical direction (y-direction).

Cyclic shearing is implemented by specifying the horizontal displacement of the element nodes along the vertical sides. The horizontal displacements of the nodes along the vertical sides are linearly distributed and the two nodes at the same height level (i.e. y coordinates are the same) have the same horizontal displacement, as demonstrated in Figure 4.2(a). This implementation of cyclic shearing closely simulates the horizontal displacement of the soil specimen in the experimental program, where the specimen is constrained by a stack of retaining rings, as shown in Figure 4.2(b). As in the experimental program, the cyclic horizontal displacement also follows a sinusoidal function.

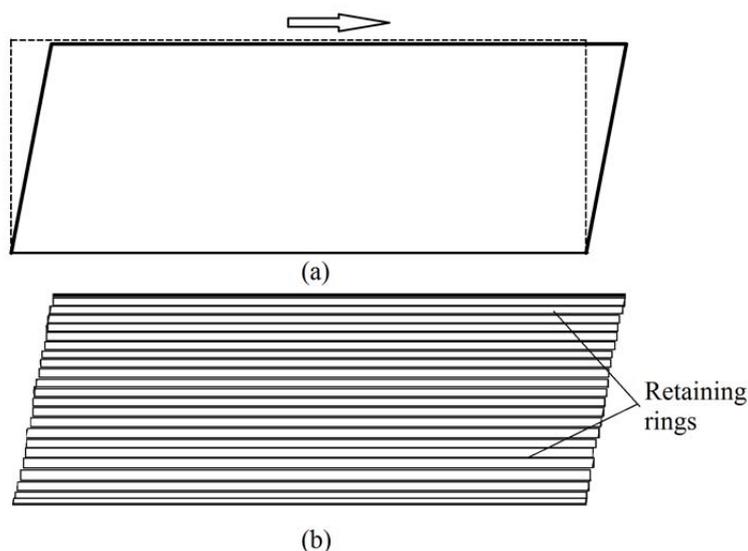


Figure 4.2 Implementation of deformation pattern in cyclic shearing in FEA model

4.1.5 Validation of the finite element model

The results of the finite element analysis with the use of actual soil properties were compared with the experimental data to validate the model. As discussed in Chapter 3, the pore pressure transducer (PPT) in the experimental program is not capable of recording real-time pore pressure level within the soil specimen and can only give an estimation of the pore pressure level at the bottom of the specimen. As a result, it is not feasible to compare the pore pressure level in the FEA with that in the experimental program to validate the model. Nevertheless, the shear stress-strain relationships in FEA and experimental results can be compared.

Figure 4.3 shows a comparison of shear stress-strain curves between the experimental and finite element analysis results. In the results presented the vertical stress is 60kPa, the shear amplitude is 10%, the shearing velocity is 0.005mm/s. Therefore, it is fully drained shearing.

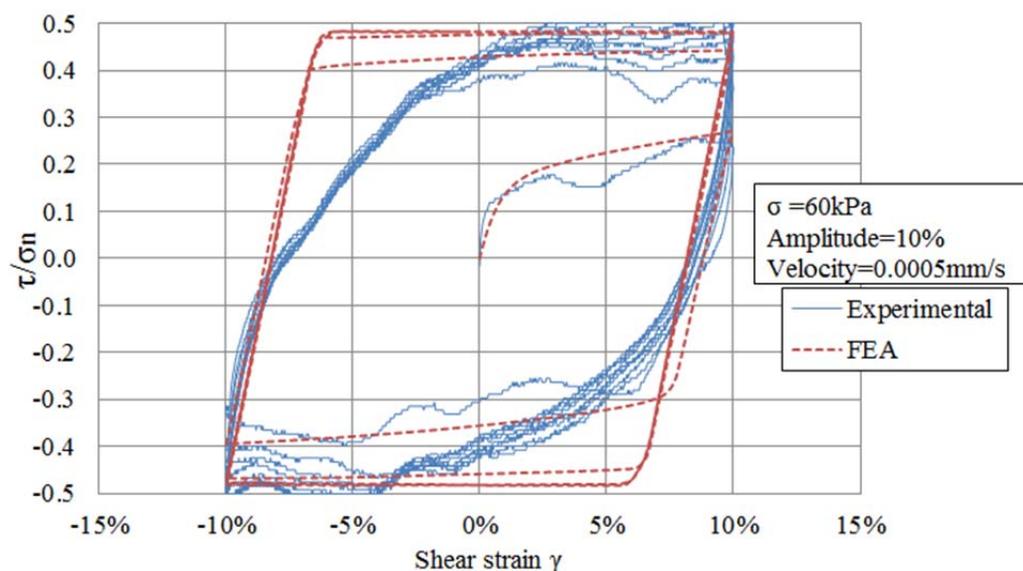


Figure 4.3 Comparison between experimental and FEA shear stress-strain curves

The peak shear stress prediction of the finite element analysis matches the experimental data closely, but there is some difference between the experimental curve and the FEA curve. While the initial portion of the shear stress-strain responses of the experimental and FEA results are close, the experimental results indicate that the slope of the curve changes continuously in subsequent shearing cycles and the curve forms an overall shape close to an ellipse, as shown in Figure 4.3. In contrast, the slope of the FEA curve remains constant before reaching the peak shear stress level, and the curve forms an overall shape close to a parallelogram. This difference can be explained by the yielding function of the modified Cam-clay model. In this model, the soil behaviour is elastic until the stress state of the soil specimen (p' , q) hits the yield surface. The shear modulus G remains unchanged after the soil reaches critical state and the slope of the FEA curve therefore remains constant for subsequent shearing cycles.

4.2 Finite element analysis results

4.2.1 Scope of the analyses

Using the finite element model developed, a parametric study was conducted to investigate the influence of vertical stress, shearing velocity and shearing amplitude on the soil behaviour in the shear zone. Table 4.3 summarises the variables in the finite element analyses (FEA). Unlike in the experimental program, where the testing equipment could not produce satisfactory horizontal load readings under very low vertical stress, a very low consolidation/vertical stress can be applied in the finite element analysis. Five shearing velocities were applied in the FEA, covering the undrained, partially drained and drained responses in cyclic shearing. Shearing amplitudes of 5% and 10% of specimen height were applied in FEA, and the height was taken as the specimen height when the consolidation stage was completed.

Table 4.3 Variables in finite element analyses

Consolidation/Vertical Stress	Shearing Velocity	Shearing Amplitude
10kPa	1.0mm/s	
20kPa	0.3mm/s	5% of H
40kPa	0.03mm/s	10% of H
60kPa	0.005mm/s	
	0.0005mm/s	

4.2.2 Consolidation stage

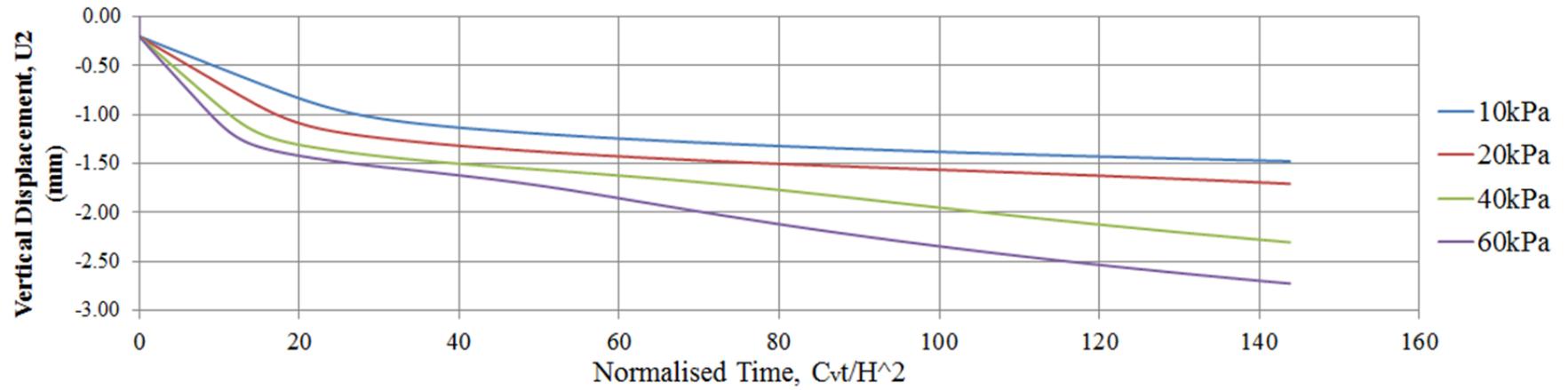
Vertical displacement

A constant vertical stress is applied at the consolidation stage and the analysis enters the cyclic shearing stage when excess pore pressure dissipation is completed. The vertical displacement vs. time and void ratio vs. time plots at the consolidation stage are presented in Figures 4.4(a) and (b) respectively. The time is normalised as $c_v t/H^2$, where c_v is the average coefficient of consolidation, taken as $2.9 \times 10^{-7} \text{ m}^2/\text{sec}$;

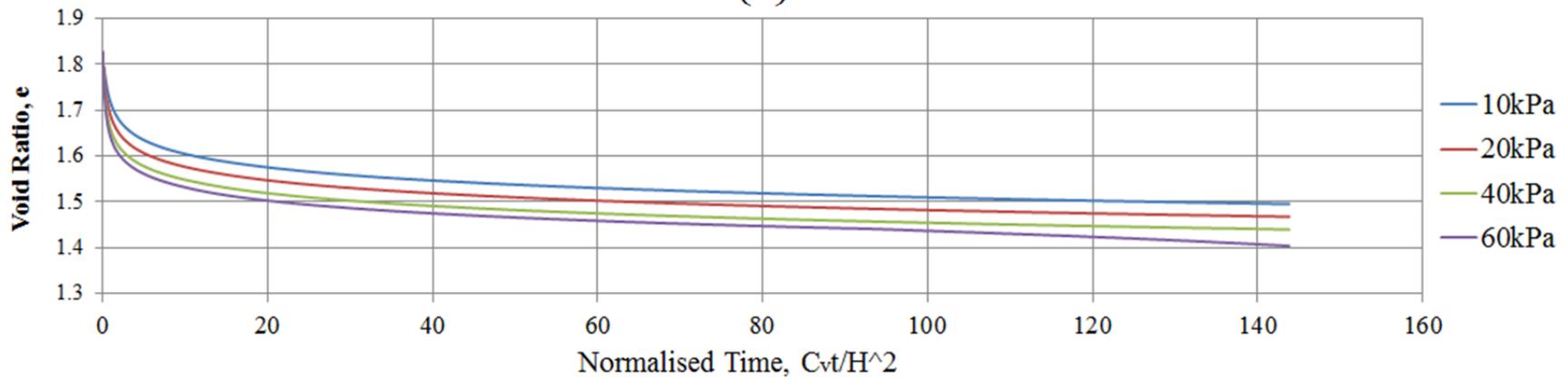
t is the elapsed time in seconds;

H is the initial soil height in meters, taken as 0.01867m. The bottom drainage port was closed during consolidation, as such one-way drainage is considered.

The results show that higher consolidation stress results in larger vertical displacement and further compression of the soil skeleton. The void ratio decreases from the initial void ratio of 1.827 to 1.5 under an applied vertical stress of 10kPa, whereas the void ratio drops from 1.827 to 1.4 under an applied vertical stress of 60kPa.



(a)



(b)

Figure 4.4 (a) Vertical displacement vs. normalised time and (b) void ratio vs. normalised time plots at consolidation stage

Excess pore pressure development

The excess pore pressure level is measured in the centre-bottom element, which is Element 20 of the model as shown in Figure 4.5 . Figure 4.6 (a) is the excess pore pressure Δu vs. normalised time plot and Figure 4.6(b) is the $\Delta u/\sigma_n$ vs. normalised time plot at consolidation stage, where σ_n is the vertical stress applied at the top surface of the soil.

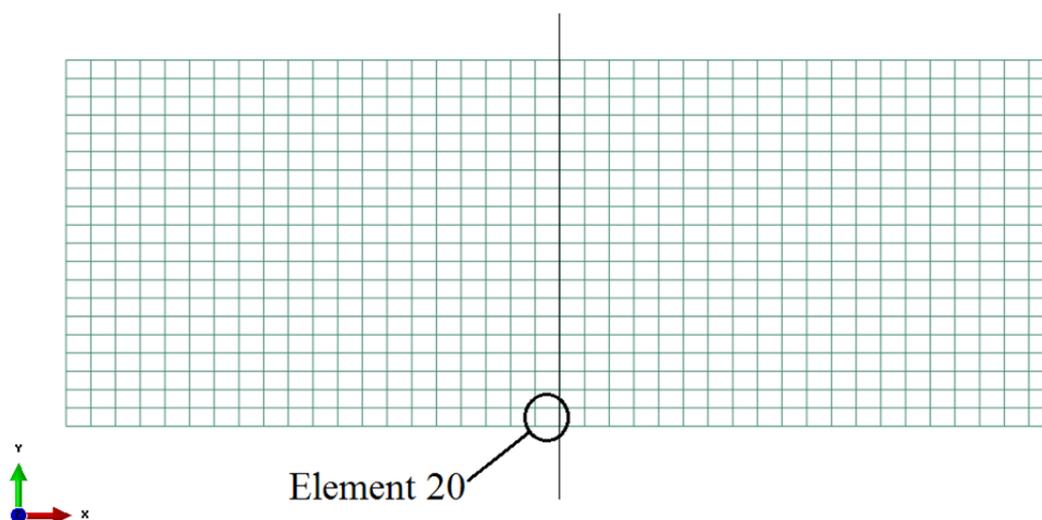
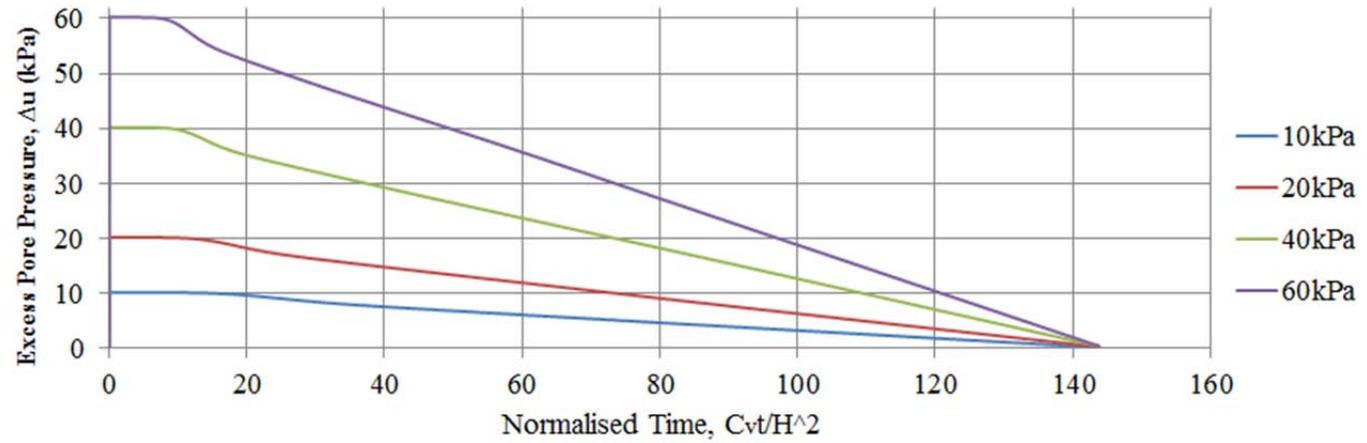
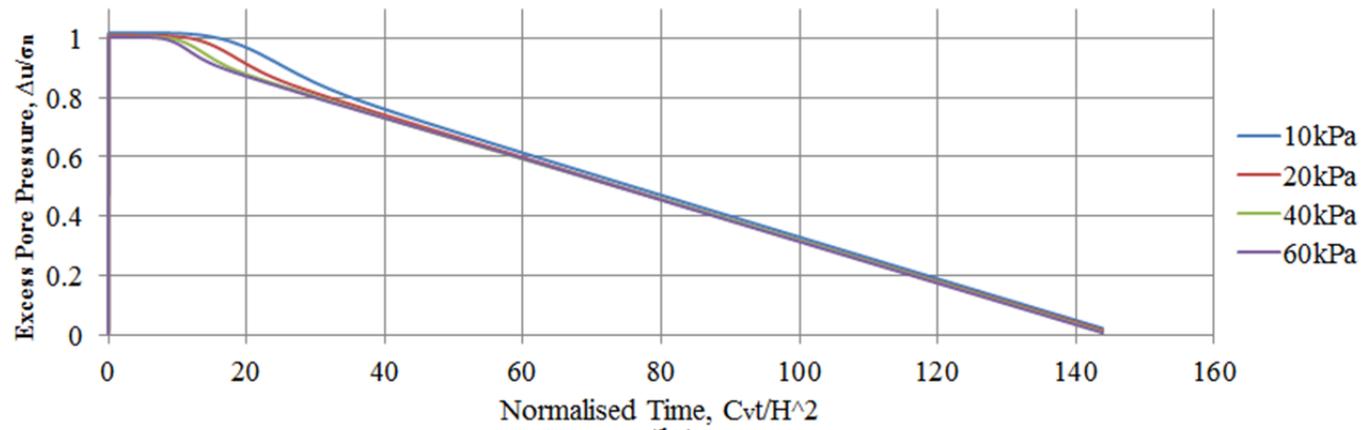


Figure 4.5 Excess pore pressure reference element

Figure 4.6 demonstrates that the excess pore pressure responses at the consolidation stage can be simulated appropriately and recorded simultaneously in the finite element analysis. As predicted, the applied vertical stress is first taken entirely by the excess pore pressure such that $\Delta u/\sigma_n = 1$. The soil skeleton gradually takes the vertical pressure and the excess pore pressure dissipates until dissipation is completed such that $\Delta u/\sigma_n = 0$.



(a)



(b)

Figure 4.6 (a) Excess pore pressure Δu vs. normalised time and (b) $\Delta u/\sigma_n$ vs. normalised time plots at consolidation stage

4.2.3 Cyclic shearing stage

As in the experimental program, the horizontal displacement follows a sinusoidal function at the cyclic shearing stage as demonstrated in Figure 4.7(a). The shear strain γ is defined as

$$\gamma = \frac{\delta}{H_{ps}} \times 100\% \quad (4.1)$$

where δ is the horizontal displacement of the top nodes in the FEA model, and H_{ps} is the height of the soil model before shearing starts, as shown in Figure 4.7(b).

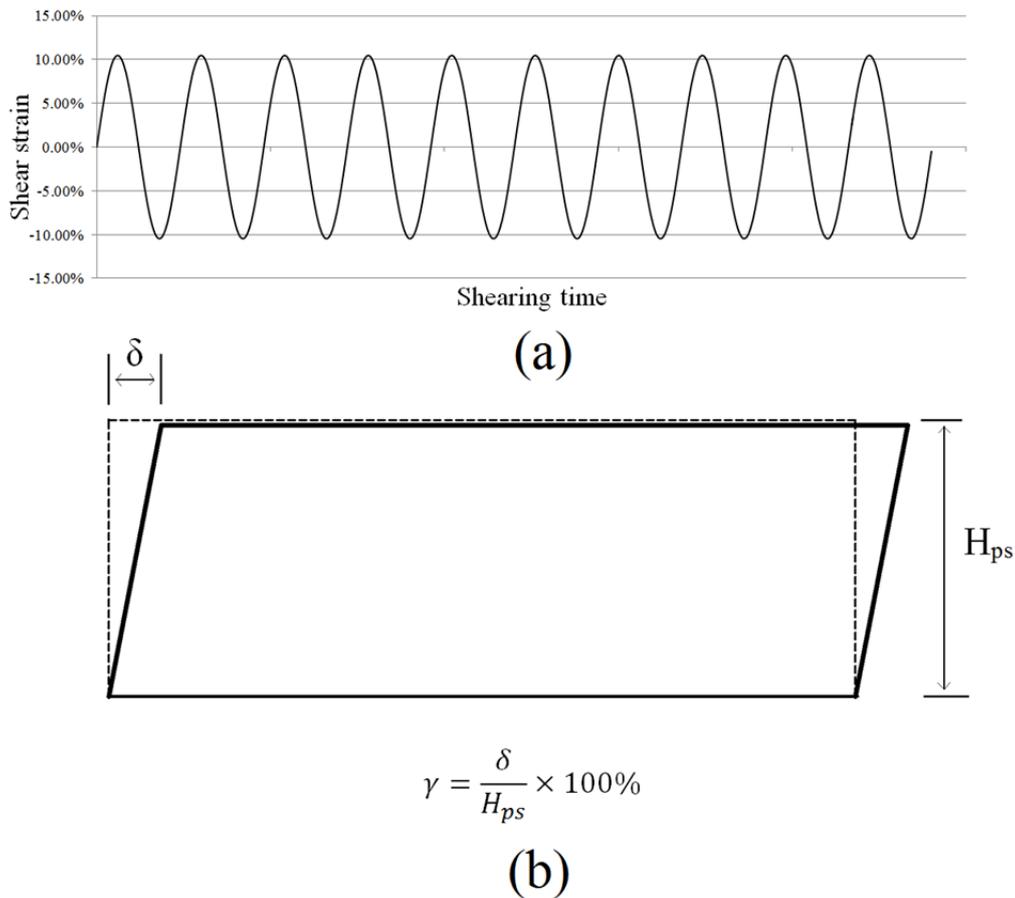


Figure 4.7 (a) horizontal displacement function and (b) shear strain definition

Pore pressure development

The pore pressure development in the soil at cyclic shearing stage was analysed and an example contour of pore pressure distribution within the soil model is shown in Figure 4.8 . In this example presented, the applied vertical stress is 60kPa, the shearing velocity is 0.3mm/s and the shearing amplitude is 5% of H.

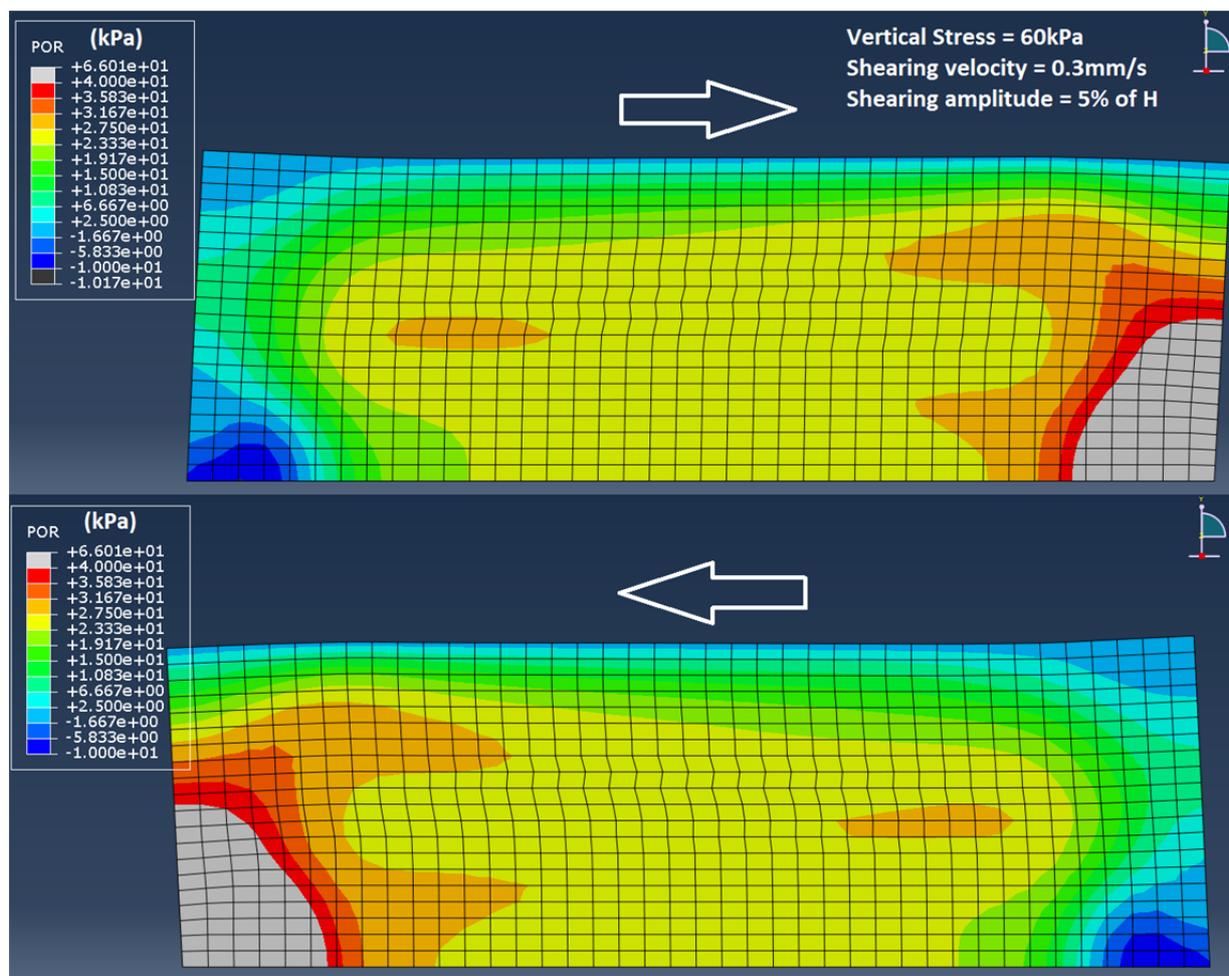


Figure 4.8 Pore pressure distribution at cyclic shearing stage

Figure 4.8 shows that the shear-induced pore pressure can be simulated in the current finite element model and the pore pressure in the soil is not uniformly distributed. Local maxima and minima are observed close to the boundaries of the soil, including the vertical sides and the corners. It is also observed that pore pressure distribution within the core of the model (middle 1/3) is relatively uniform and the excess pore pressure level is again measured in the centre-bottom element, which is Element 20 of the model, as shown in Figure 4.5 .

The pore pressure versus time plots are presented in Figures 4.9 and 4.10 for applied vertical stresses of 10kPa and 60kPa respectively. The shear strain versus time curves are also plotted

on the same figures. The pore pressure versus time plots for applied vertical stresses of 20kPa and 40kPa are included in Appendix B.

As elaborated in Chapter 3, the pore pressure transducer readings in the experimental program can only provide an estimation of the pore pressure level in the soil specimen, whereas the exact pore pressure within the soil can be obtained using finite element analysis. Nevertheless, the pore pressure development in the finite element analyses is consistent with the pore pressure development in the experimental results.

At high shearing velocities (1mm/s & 0.3mm/s), excess pore pressure continuously builds up as cyclic shearing progresses, and induces a higher level of excess pore pressure. Therefore it is undrained shearing.

At moderate shearing velocities (0.03mm/s & 0.005mm/s), excess pore pressure is generated and peaks after reaching a moderate level. As cyclic shearing continues, excess pore pressure dissipates eventually. Therefore it is partially drained shearing.

At very low shearing velocities (0.0005mm/s & 0.0001mm/s), excess pore pressure does not build up and therefore it is drained shearing.

For an applied vertical stress σ_n of 10kPa, the maximum excess pore pressure Δu generated is close to 2kPa, which gives $\Delta u/\sigma_n$ close to 0.2. However, for applied vertical stresses σ_n of 20kPa, 40kPa and 60kPa, the maximum excess pore pressure Δu generated is close to $\Delta u/\sigma_n = 0.5$.

In Figures 4.9 and 4.10, when comparing the excess pore pressure development of analyses that have the same vertical stress and shearing velocities but vary in amplitude (5%, 10%), it is clear that shear amplitude has little or no effect on the generation of excess pore pressure during cyclic shearing. This finding is consistent with the suggestion made by White et al. (2011b), who argue that the mobilisation of axial resistance in offshore pipelines is a time-related process rather than being linked to the distance of shearing.

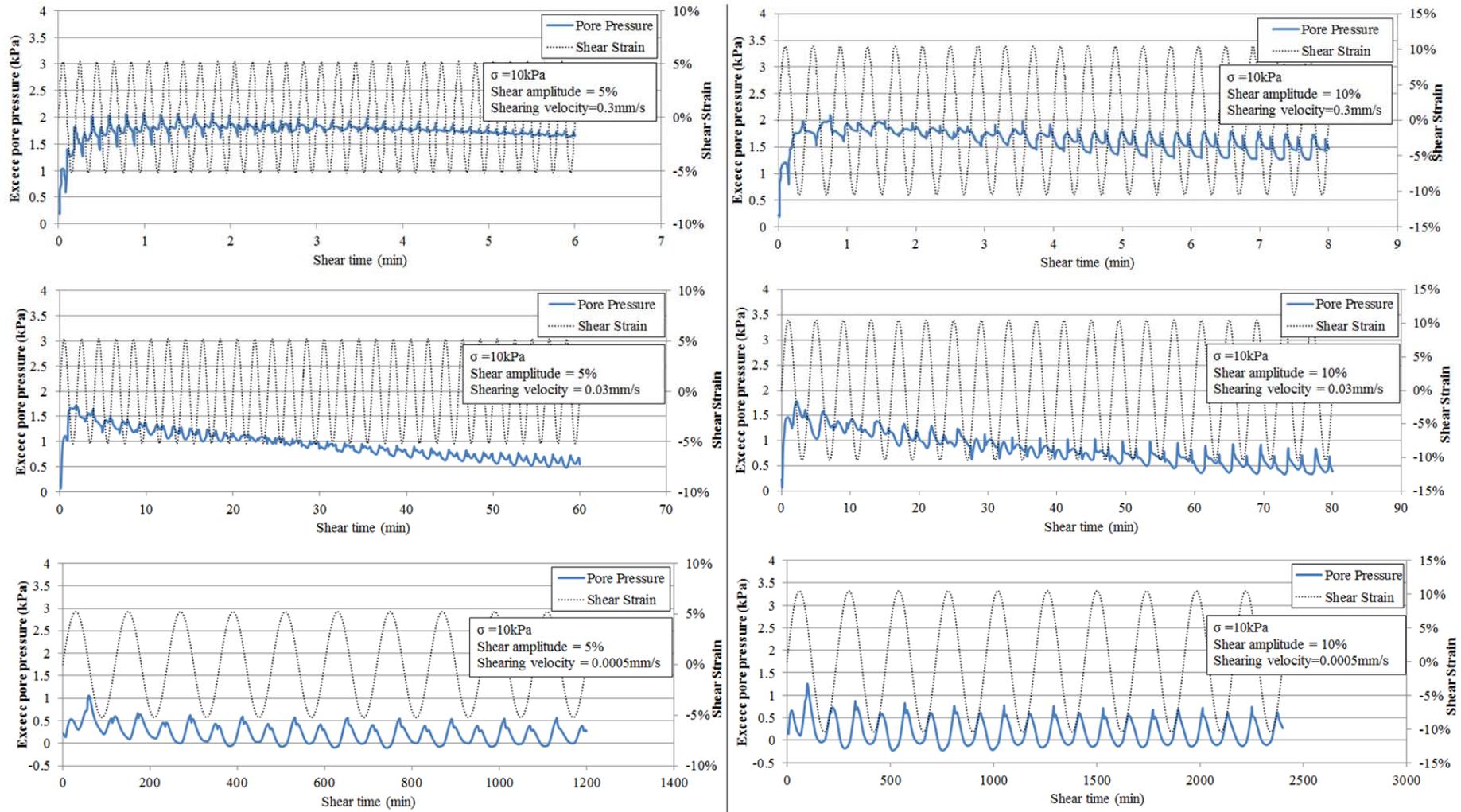


Figure 4.9 Excess pore pressure development at cyclic shearing stage for applied vertical stress of 10kPa

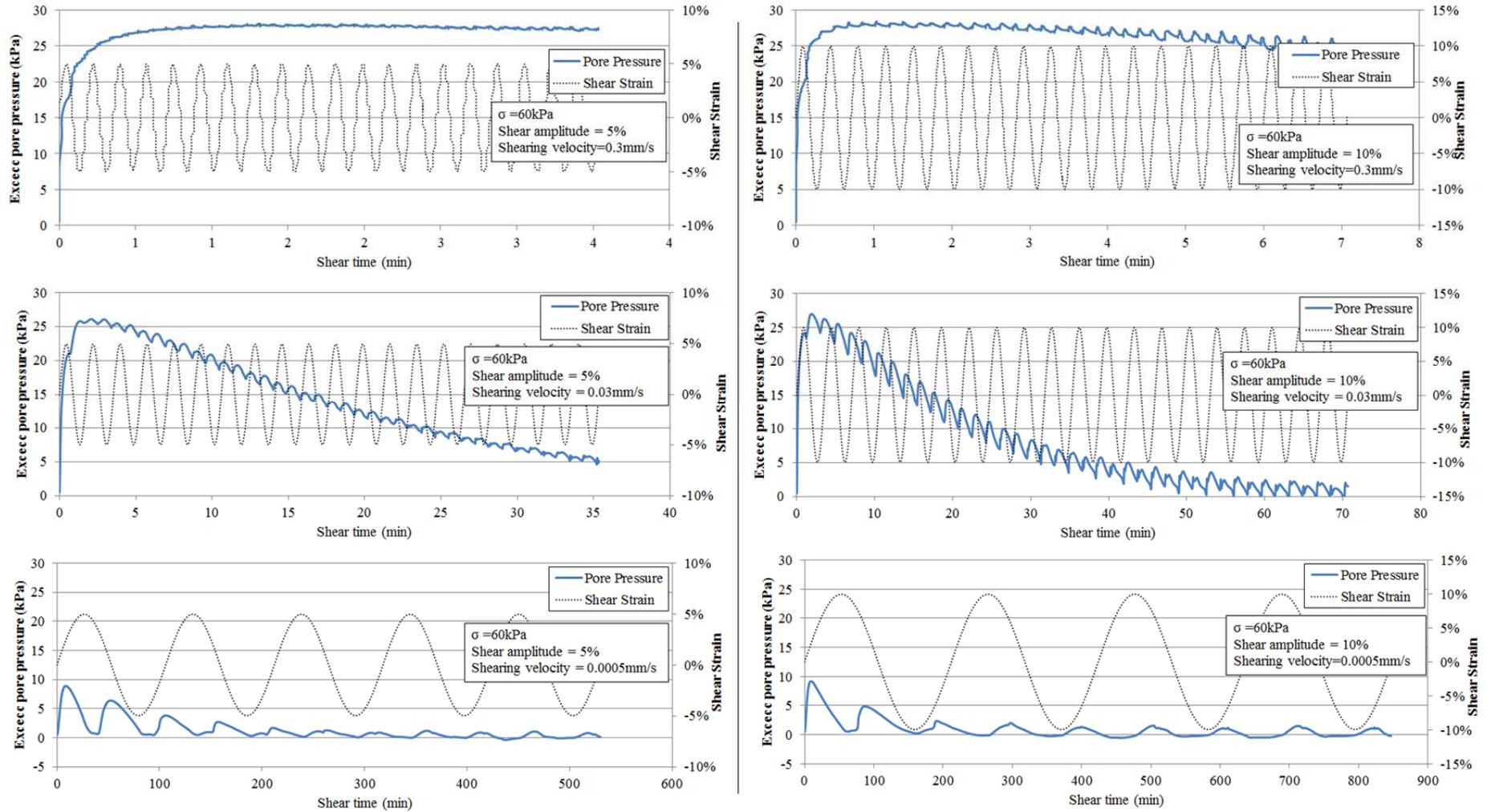


Figure 4.10 Excess pore pressure development at cyclic shearing stage for applied vertical stress of 60kPa

Continuous consolidation

Since the pore water is free to drain from the top surface of the soil model, continuous consolidation takes place at the cyclic shearing stage in the finite element analyses. This is consistent with what was observed in the experimental results. The void ratios versus shear strain plots at the cyclic shearing stage are presented in Figures 4.11 and 4.12 for applied vertical stresses of 10kPa and 60kPa respectively. The void ratios versus shear strain plots at the cyclic shearing stage for applied vertical stresses of 20kPa and 40kPa are included in Appendix B. The void ratio is measured from Element 20 of the model as shown in Figure 4.5 and the analysis results with a shear amplitude of 10% of H are presented.

At high shearing velocities (1mm/s & 0.3mm/s), the decrease in void ratio is minimal because excess pore pressure continuously builds up and it is undrained shearing. At moderate shearing velocities (0.03mm/s & 0.005mm/s), the void ratio drops continuously as excess pore pressure dissipates and the soil skeleton compresses further. As cyclic shearing progresses, the void ratio converges as the dissipation of excess pore pressure is completed and there is no further compression. At very slow shearing velocity (0.0005mm/s & 0.0001mm/s), most of the compression of the soil skeleton and the decrease in void ratio occur in the first two cycles, and there is no further change in void ratio in subsequent cycles.

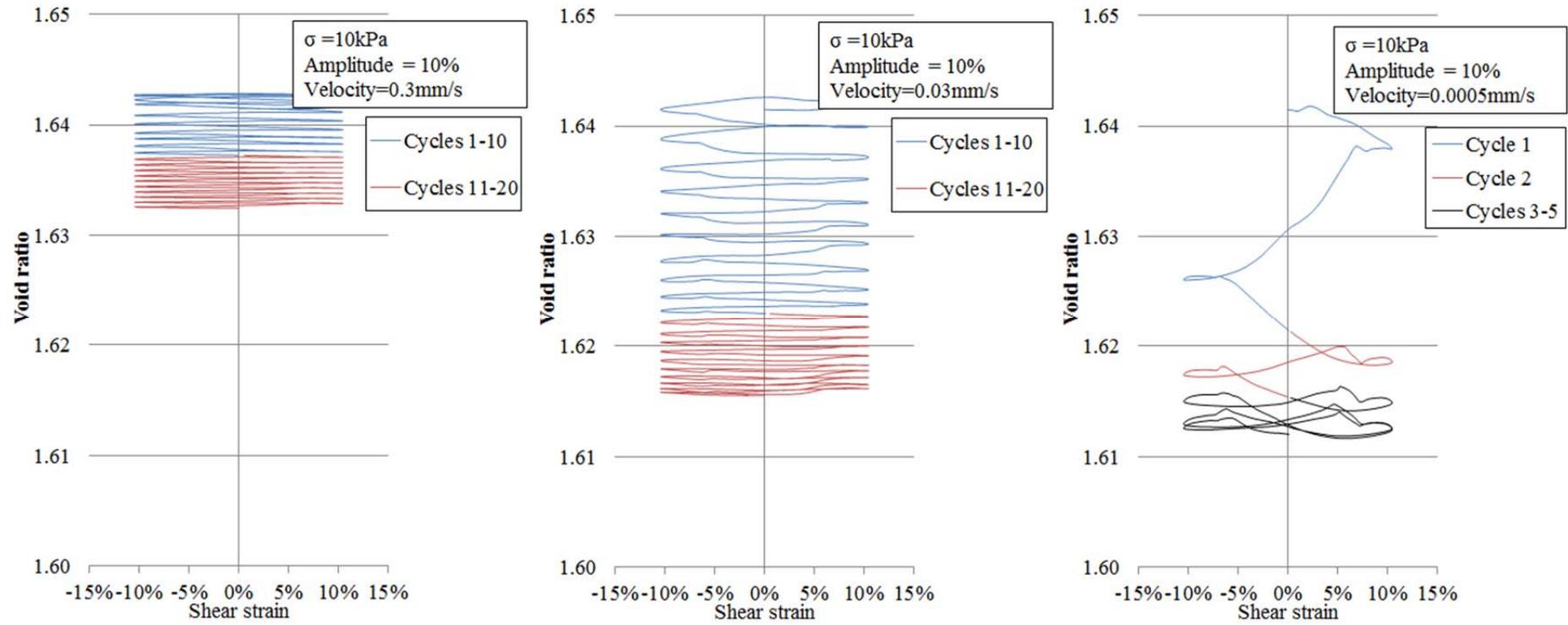


Figure 4.11 Void ratio versus shear strain plots for applied vertical stress of 10kPa and shear amplitude of 10% of H

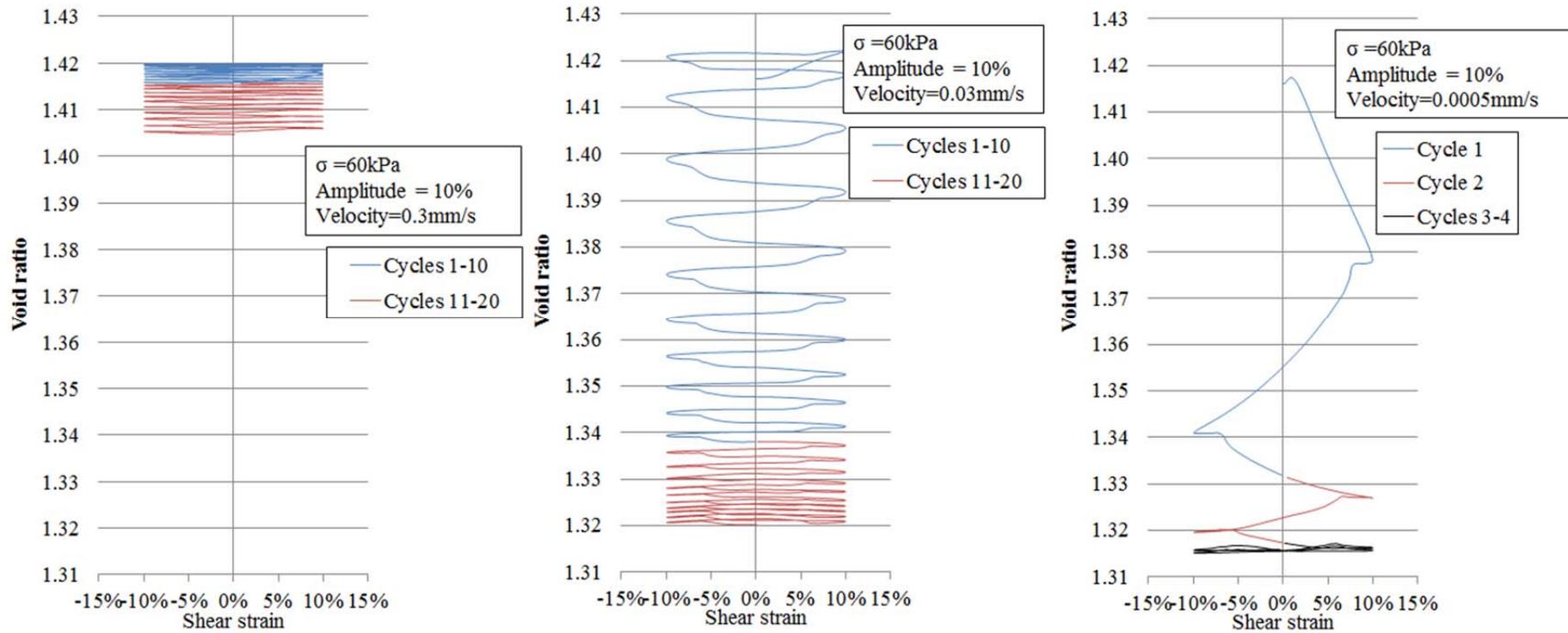


Figure 4.12 Void ratio versus shear strain plots for applied vertical stress of 60kPa and shear amplitude of 10% of H

Shear stress-strain relationship

The shear stress-strain curves at the cyclic shearing stage are plotted on Figures 4.13 and 4.14 for applied vertical stresses of 10kPa and 60kPa respectively. The shear stress-strain curves at the cyclic shearing stage curves for applied vertical stresses of 20kPa and 40kPa are provided in Appendix B. The level of shear stress is normalised as τ/σ_n , where τ is the shear stress and σ_n is the applied vertical stress at the top surface.

At high shearing velocities (1mm/s & 0.3mm/s), the shearing is undrained and τ/σ_n remains unchanged as cyclic shearing progresses. For a very low applied vertical stress of 10kPa, the undrained shear resistance equals $\tau/\sigma_n=0.4$, while for applied vertical stresses of 20kPa, 40kPa and 60kPa, the undrained shear resistance gives $\tau/\sigma_n=0.25$. This difference can be explained by the level of excess pore pressure generated under a vertical stress of 10kPa. For an applied vertical stress σ_n of 10kPa, the maximum excess pore pressure Δu generated is close to 2kPa, which gives $\Delta u/\sigma_n$ close to 0.2. However, for applied vertical stresses σ_n of 20kPa, 40kPa and 60kPa, the maximum excess pore pressure Δu generated is close to $\Delta u/\sigma_n = 0.5$.

At moderate shearing velocities of 0.03mm/s and 0.005mm/s, the shearing is partially drained and τ/σ_n increases as cyclic shearing progresses. This is because excess pore pressure dissipates and effective stress increases, resulting in higher shear resistance. The initial shear resistance is close to the undrained shear resistance ($\tau/\sigma_n=0.4$ for $\sigma_n=10$ kPa and $\tau/\sigma_n=0.25$ for $\sigma_n=20,40,60$ kPa). As cyclic shearing continues, the shear resistance eventually reaches a peak level close to $\tau/\sigma_n=0.45$. Shearing amplitude has no effect on the peak resistance level.

At low shearing velocities of 0.0005mm/s and 0.0001mm/s, the shearing is drained and fully drained shear resistance is reached within 4 cycles for 5% shear amplitude and 2 cycles for 10% shear amplitude. The fully drained shear resistance is at a level close to $\tau/\sigma_n=0.48$.

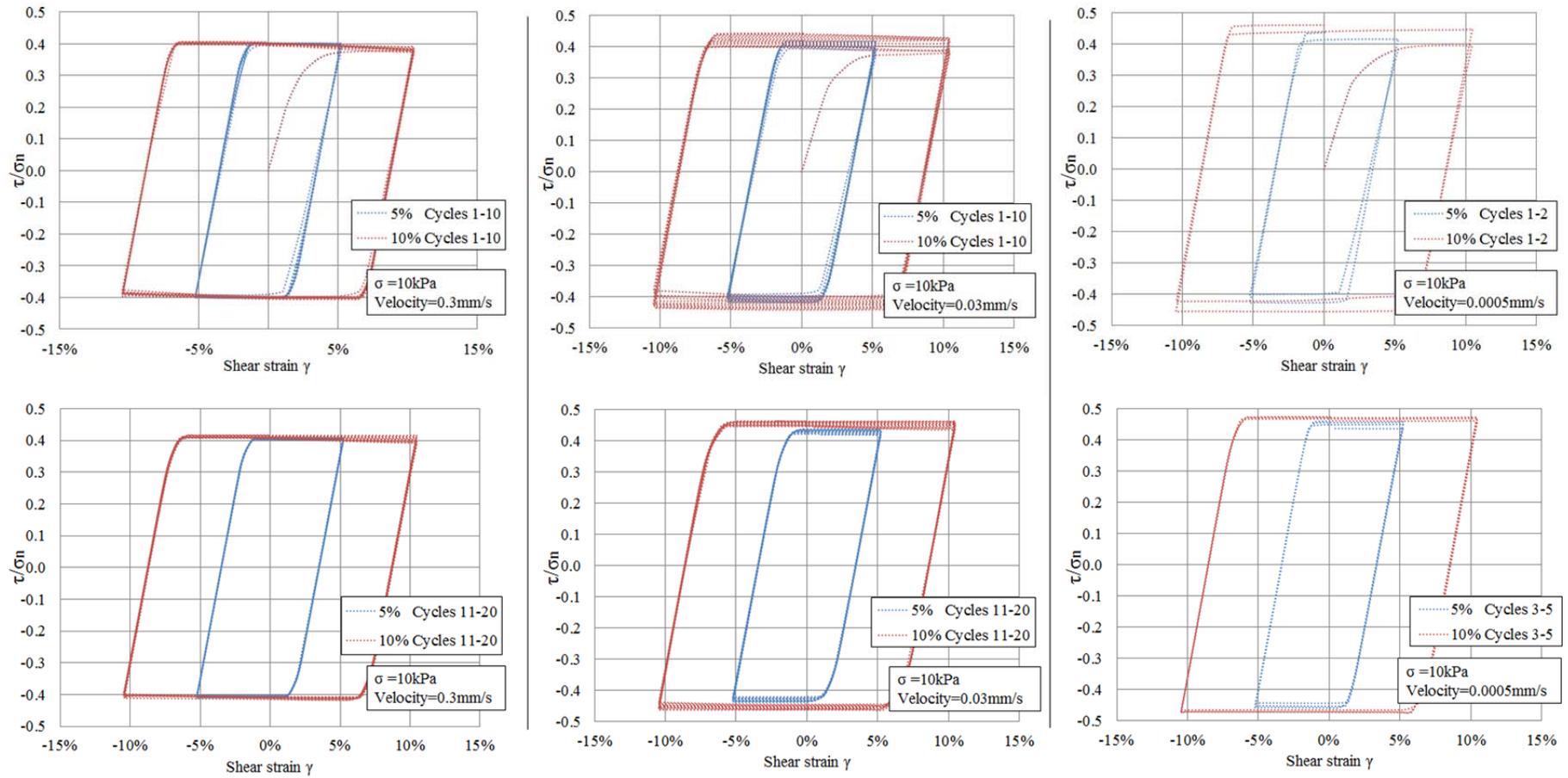


Figure 4.13 Shear stress-strain curves for applied vertical stress of 10kPa

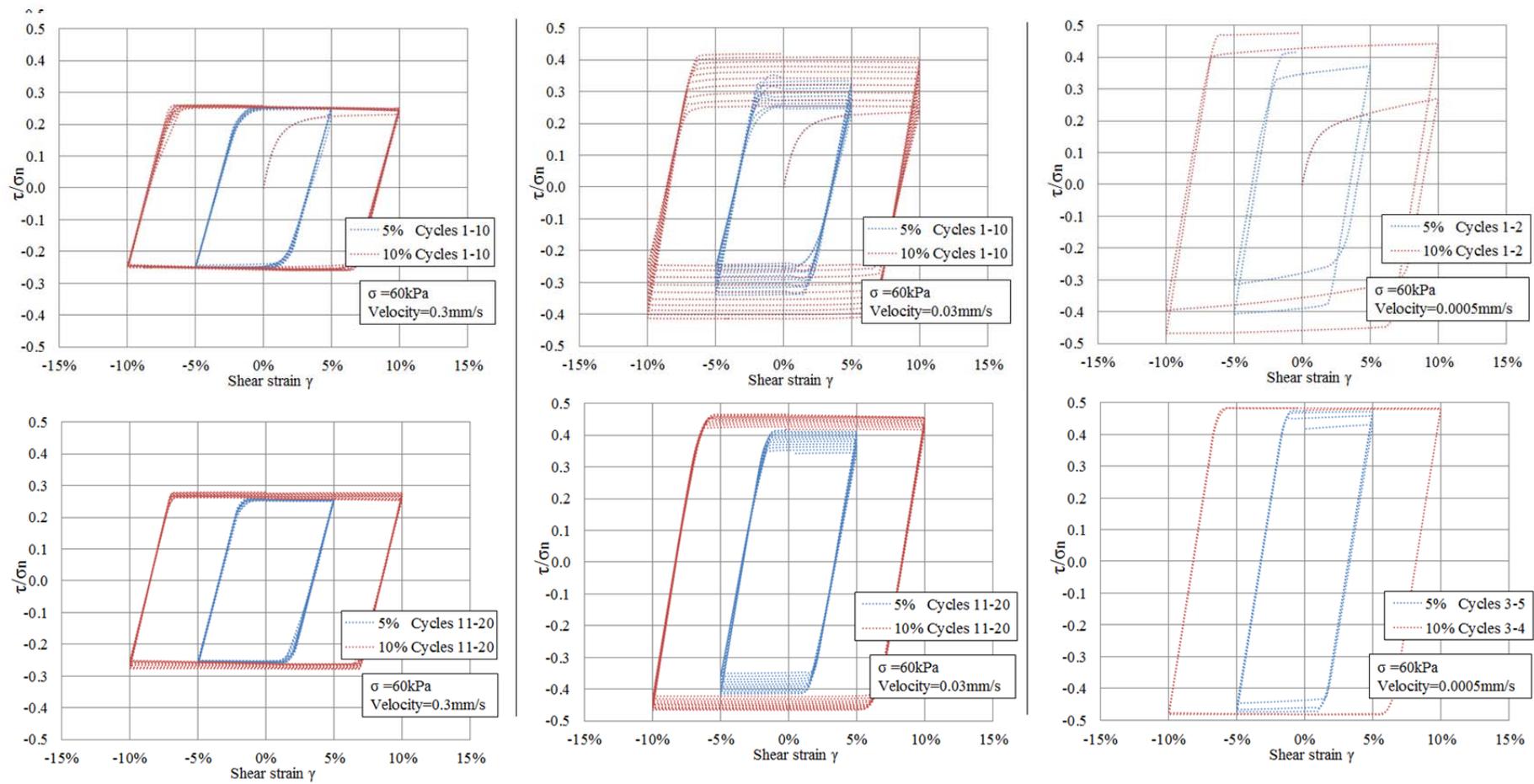


Figure 4.14 Shear stress-strain curves for applied vertical stress of 60kPa

Effective stress interpretation

Finite element analysis is capable of providing effective stress levels at the cyclic shearing stage and the shear stress-strain relationship can therefore be interpreted in terms of effective stress. Figure 4.15 presents the effective stress interpretation of the shear stress-strain relationship of applied vertical stresses of 10kPa and 60kPa with shear amplitude of 10%. τ/σ'_n is plotted against γ , where τ is the shear stress, σ'_n is the effective vertical stress and γ is the shear strain.

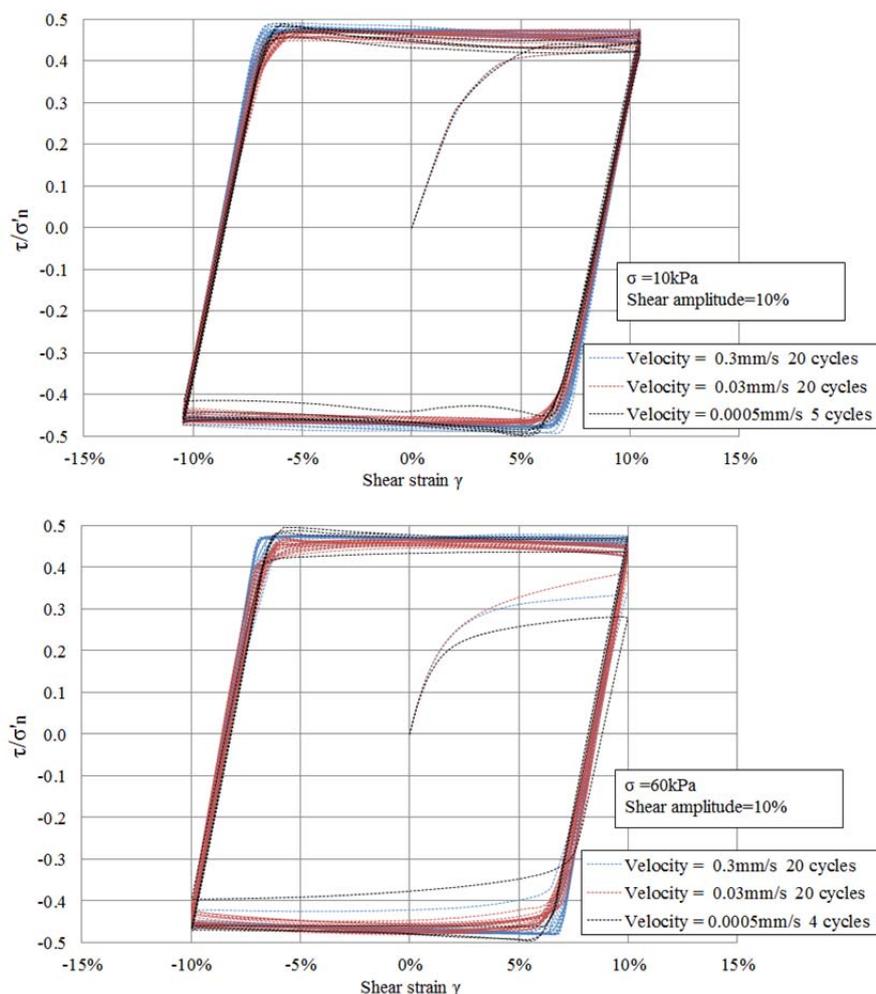


Figure 4.15 Effective stress interpretation of shear stress-strain relationship

In the effective stress interpretation, all shear stress-strain curves overlie each other, except for the initial responses. This is true for both cycles at a particular shearing velocity, and cycles at different shearing velocities. This verifies that the axial shear resistance in the shear zone is dominated by the excess pore pressure levels. The effective stress interpretation also suggests that, despite the fact that the experimental set-up cannot produce satisfactory output

at a low vertical stress (i.e. 10kPa), the application of a higher vertical stress (i.e. 60kPa) can still capture the appropriate soil behaviour in cyclic shearing.

4.3 Summary of major findings

Finite element analyses were conducted to capture the behaviour of soil within the shear zone, using the modified Cam-clay model. A two-dimensional finite element model has been developed and analyses were performed applying varying vertical stresses, shearing velocities and shearing amplitudes. The soil behaviour in the shear zone, in particular the shear-induced pore pressure, can be simulated using the current finite element model.

For an applied vertical stress σ_n of 10kPa, the maximum excess pore pressure Δu generated is close to 2kPa, which gives $\Delta u/\sigma_n$ close to 0.2. However, for applied vertical stresses σ_n of 20kPa, 40kPa and 60kPa, the maximum excess pore pressure Δu generated is close to $\Delta u/\sigma_n = 0.5$. When comparing the excess pore pressure development of analyses that have the same vertical stress and shearing velocities but vary in amplitude (5%, 10%), it was found that shear amplitude has little or no effect on the generation of excess pore pressure during cyclic shearing. This finding is consistent with the suggestion made by White et al. (2011b), who argue that the mobilisation of axial resistance in offshore pipelines is a time-related process rather than being linked to the distance of shearing.

Since the pore water is free to drain from the top surface of the soil model, continuous consolidation takes place at the cyclic shearing stage in the finite element analyses, which is consistent with what was observed in the experimental results.

Figure 4.16 presents the residual shear resistance versus velocity plot of the finite element analyses completed. The level of shear resistance is presented as τ/σ_{total} , where τ is the shear/horizontal stress and σ_{total} is the total normal stress applied. The shearing velocity is normalised as vH/c_v , where

v is the shearing velocity in m/sec;

H is the soil specimen height before shearing;

c_v is the average coefficient of consolidation, taken as 2.9×10^{-7} m²/sec ;

The residual resistance levels versus velocity plots of the experimental program are also presented in Figure 4.16 .

The undrained shear resistance gives $\tau/\sigma_n=0.4$ for a very low applied vertical stress of 10kPa, while for applied vertical stresses of 20kPa, 40kPa and 60kPa, the undrained shear resistance level is close to $\tau/\sigma_n=0.25$. This is because under a very low vertical stress of 10kPa the maximum excess pore pressure Δu generated gives $\Delta u/\sigma_n$ close to 0.2, whereas for vertical stresses σ_n of 20kPa, 40kPa and 60kPa, the maximum excess pore pressure Δu generated is close to $\Delta u/\sigma_n = 0.5$. In comparison, the experimental results indicate an undrained residual resistance of $\tau/\sigma_n=0.17$. The level of fully drained shear resistance is consistent under different applied vertical stresses, being close to $\tau/\sigma_n=0.48$. In comparison, the experimental results indicate a fully drained shear resistance of $\tau/\sigma_n=0.52$. The difference in shear resistance levels between the finite element analysis and the experimental program may be attributed to the limitations of the experimental equipment and some disparities in the drainage conditions.

Figure 4.16 also identifies the drained and undrained limits of the residual resistance, and the distribution of residual resistance against normalised velocity is uniform.

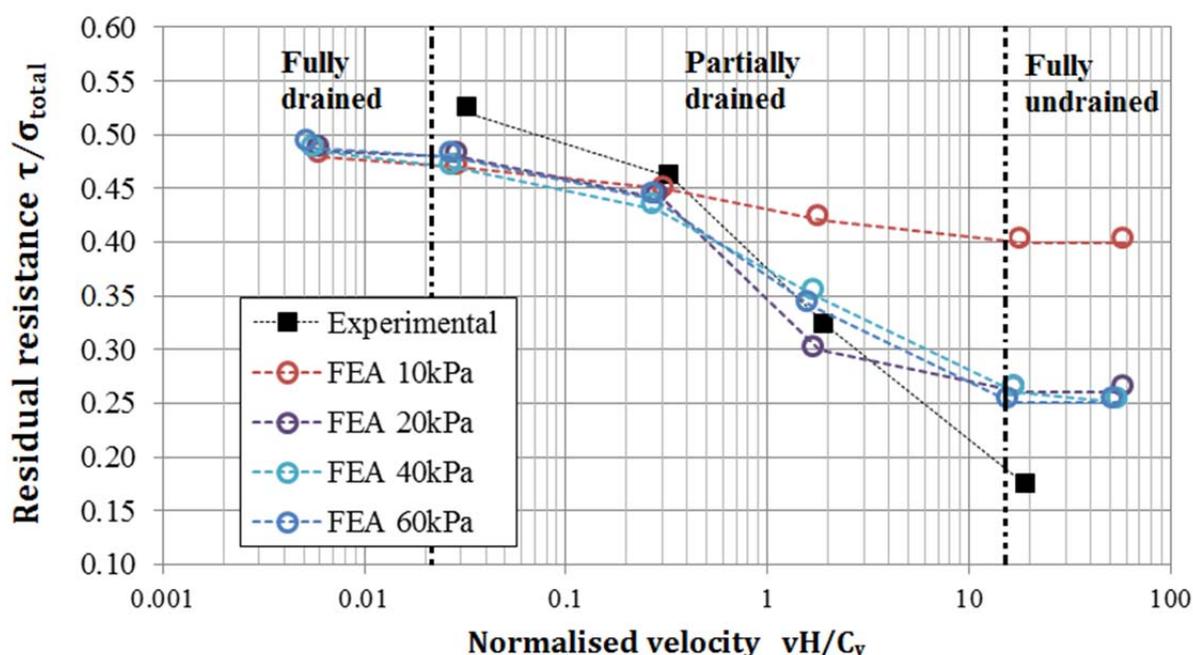


Figure 4.16 Residual resistance versus normalised velocity

In the effective stress interpretation, all shear stress –strain curves overlie each other, except for the initial responses. This is true for both cycles at a particular shearing velocity, and

cycles at different shearing velocities. This verifies that the axial shear resistance in the shear zone is dominated by the excess pore pressure levels. The effective stress interpretation also suggests that, despite the fact that the experimental set-up cannot produce satisfactory output at a low vertical stress (i.e. 10kPa), the application of a higher vertical stress (i.e. 60kPa) can still capture the appropriate soil behaviour in cyclic shearing.

Chapter 5 – Conclusions and recommendations

This thesis presents the results of research on the behaviour of soil within the shear zone in pipe axial walking problems. Both experimental and numerical modelling studies were undertaken. This chapter summarises the major findings of the present research, and provides recommendations for future research.

5.1 Conclusions

Offshore pipeline axial walking is the undesired accumulation of axial displacement of offshore pipelines during operation cycles. Research indicates that the axial resistance will tend towards the drained values during cycles of pipe movement, regardless of the rate or duration of each movement. A reliable prediction of this mechanism could bring significant design benefits, as drained resistance is usually higher than undrained resistance. Consequently, a higher range of axial resistance can be applied, leading to more cost-effective design. The pipe-soil interaction in pipe axial walking problems is extensively influenced by the soil response. The axial response is significantly influenced by the generation and dissipation of excess pore water pressure which leads to undrained and drained soil behaviour. It is likely that the response is between fully undrained and fully drained in typical field conditions. Analysis of the field data also suggests that the mobilisation of axial resistance is a time-related process instead of being linked to the distance of shearing. This suggestion is consistent with excess pore pressure development in cycles. In other words, the full pipe-soil resistance is only mobilised when steady pore pressures are reached. The residual resistance is the primary component of the load-displacement relationship in pipe axial walking, and it is dominated by the axial displacement rate.

The portion of the soil below the pipe which undergoes shearing, characterised as the *shear zone*, is significant for pipe axial walking assessment. Cyclic direct simple shear testing was selected as the preferred testing method to conduct the investigation owing to its various merits, including its loading pattern, allowing rotation of principle stress axes, the small specimen size required, and its shear failure planes. Furthermore, a finite element model has

been developed which can capture the behaviour of soil within the shear zone, utilising an advanced constitutive soil model.

Both experimental and numerical analysis results suggest that at a high shearing velocity, excess pore pressure builds up continuously in the shear zone and the shearing is undrained. At a moderate shearing velocity, excess pore pressure builds up and dissipates, and effective stress drops and rises, eventually resulting in a drained shear resistance. At a low shearing velocity, no excess pore pressure builds up and the shearing is drained.

The relationship between residual shear resistance and shearing velocity has been established and the drained and undrained limits of the residual resistance have been identified in cyclic direct simple shear tests. Some shifting factors or transferring equations may be developed to relate the residual resistance as well as the undrained/drained limits in direct simple shear tests to those in pipe-soil interface interactions in the axial direction. If standard test methods such as the current direct simple shear test can be applied in the practical design, taking offshore pipeline axial walking into consideration, the process would be significantly simplified in terms of time and cost.

5. 2 Recommendations for future research

Some modifications to the standard direct simple shear devices are required if direct simple shear testing is to be widely used in practical design taking offshore pipeline axial walking into account. First and foremost, since the effective stresses in axial walking problems are very low, the device should be capable of providing reliable load readings under very low stress levels (2kPa to 10kPa). In addition, although the truly undrained condition is not required in such tests, it is desirable that the real-time pore water pressure level in the specimen can be accurately measured.

A more complicated three-dimensional finite element model can be developed based on the current two-dimensional model, if convergence issues can be resolved. The data obtained from both experimental work and numerical analysis can be applied in the calibration of large-scale pipe axial walking modelling, and provide guidance on the design of offshore pipelines considering on-bottom stability in axial direction.

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Appendices

Appendix A Critical state soil theory and modified Cam-clay model

Introduction

The specific model implemented in Abaqus Standard is an extension of the Modified Cam-clay model (Abaqus 6.13 Online Documentation). The model is based on the critical state soil theory developed by researchers at Cambridge University (Roscoe et al., 1958, Burland and Roscoe, 1969, Schofield and Wroth, 1968). The model is capable of describing the stress-strain behaviour of soft soils. In particular, the model can predict the pressure-dependent soil strength and the compression and dilatancy (volume change) caused by shearing. Because the model is based on critical state theory, it predicts unlimited soil deformations without changes in stress or volume when the critical state is reached.

The Cam-clay model assumes that the soil is fully saturated. When the soil is loaded, significant irreversible (plastic) volume changes occur, due to the water that is expelled from the voids. In critical state soil theory, the state of a soil specimen is characterised by three parameters: mean effective stress p' , deviator stress (shear stress) q , and void ratio e . The mean effective stress can be defined in terms of the principal effective stresses σ'_1 , σ'_2 , σ'_3 as

$$p' = \frac{1}{3}(\sigma'_1 + \sigma'_2 + \sigma'_3) \quad (\text{A.1})$$

and the shear stress is defined as

$$q = \frac{1}{\sqrt{2}}\sqrt{(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_1 - \sigma'_3)^2} \quad (\text{A.2})$$

For the consolidation stage of a consolidated-drained triaxial compression test, $\sigma'_1 = \sigma'_2 = \sigma'_3$, where σ'_3 is the confining pressure, thus

$$p' = \frac{1}{3}(\sigma'_1 + \sigma'_2 + \sigma'_3) = \sigma'_3 \quad (\text{A.3})$$

and

$$q = 0 \quad (\text{A.4})$$

For the shearing stage of a triaxial compression test $\sigma'_1 \neq \sigma'_2 = \sigma'_3$, therefore

$$p' = \frac{1}{3}(\sigma'_1 + \sigma'_2 + \sigma'_3) = \frac{1}{3}(\sigma'_1 + 2\sigma'_3) \quad (\text{A.5})$$

And

$$q = \frac{1}{\sqrt{2}}\sqrt{2(\sigma'_1 - \sigma'_3)^2} = \sigma'_1 - \sigma'_3 \quad (\text{A.6})$$

The effective stress path of a triaxial test represents the locus of the effective stress state in the p' - q plane. For a consolidated-drained triaxial test, the effective stress path is a straight line and the slope of the line is defined as $\Delta q/\Delta p'$.

Because σ'_3 is constant and from Equation A.6

$$\Delta q = \Delta\sigma'_1 - \Delta\sigma'_3 = \Delta\sigma'_1 - 0 = \Delta\sigma'_1$$

And from Equation A.5

$$\Delta p' = \frac{1}{3}(\Delta\sigma'_1 + \Delta 2\sigma'_3) = \frac{1}{3}(\Delta\sigma'_1 + 0) = \frac{\Delta\sigma'_1}{3}$$

Therefore

$$\text{slope} = \frac{\Delta\sigma'_1}{\Delta\sigma'_1/3} = 3$$

Normal consolidation line and Unloading-reloading lines

A typical e - $\log p'$ curve of an isotropic consolidation test is shown in Figure A.1(a). The maximum past pressure exerted on the clay specimen is defined as the pre-consolidation pressure p'_c , and a normally consolidated (NC) clay is defined as a clay that has a present vertical effective stress p'_0 equal to its pre-consolidation pressure p'_c . An over-consolidated (OC) clay is defined as a clay that has a present vertical effective stress less than its pre-consolidation pressure. The over-consolidation ratio is the ratio of the pre-consolidation pressure to the present vertical effective stress ($\text{OCR} = p'_c / p'_0$).

The pre-consolidation pressure is located near the point where the e - $\log p'$ curve changes its slope. The compression index (C_c) and swelling index (C_s) can also be obtained from the e -

$\log p'$ curve. The compression index is the slope of the loading portion in the e - $\log p'$ plane, and the swelling index is the slope of the unloading portion.

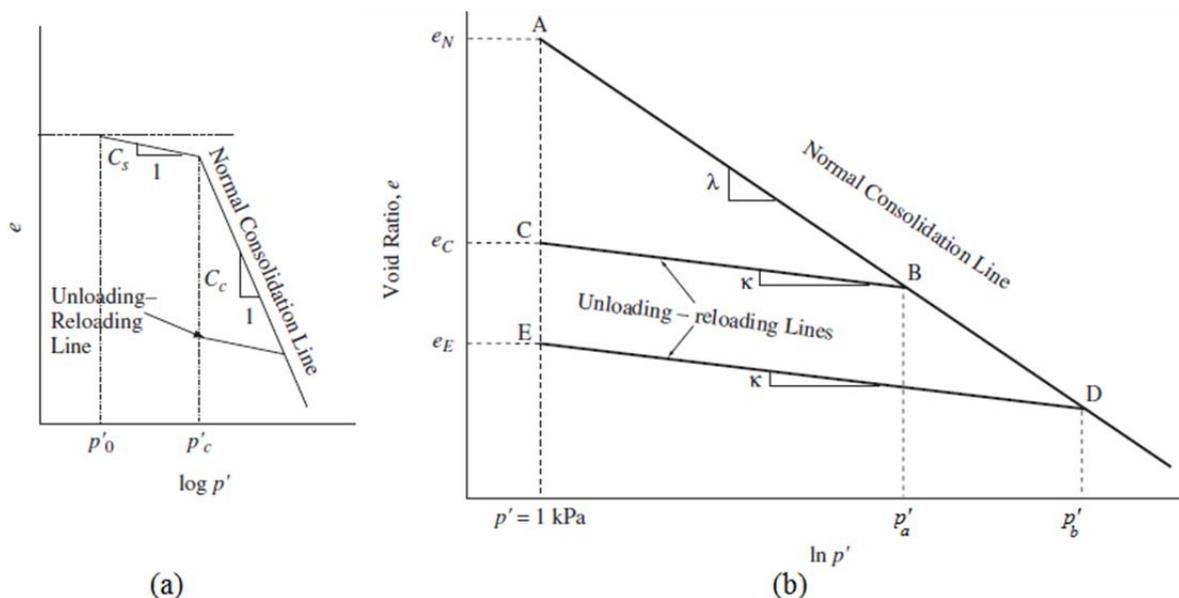


Figure A.1 (a) e - $\log p'$ curve of an isotropic consolidation test and (b) consolidation curve in the e - $\ln p'$ plane (Helwany, 2007)

In the derivation of the modified Cam-clay model, it is assumed that when a soil sample is consolidated under isotropic stress conditions ($p' = \sigma'_1 = \sigma'_2 = \sigma'_3$), the relationship between its void ratio (e) and $\ln p'$ is a straight line. This line is the normal consolidation line shown in Figure A.1(b). In addition, there exists a set of straight unloading-reloading (swelling) lines that describe the unloading-reloading behaviour of the soft soil in the e - $\ln p'$ plane. The plane λ is the slope of the normal consolidation line and κ is the slope of the unloading-reloading line.

In the e - $\ln p'$ plane, the normal consolidation line is defined by the equation

$$e = e_N - \lambda \ln p' \tag{A.7}$$

and the equation for an unloading-reloading line has the form

$$e = e_c - \kappa \ln p' \tag{A.8}$$

The material parameters λ , κ , and e_N are unique for a particular soil, and e_N is the void ratio on the normal consolidation line at unit mean effective stress (point A in Figure A.1b).

Slopes λ and κ of the normal consolidation and unloading-reloading lines in the $e-\ln p'$ plane are related to the compression index (C_c) and swelling index (C_s):

$$\lambda = \frac{C_c}{\ln 10} = \frac{C_c}{2.3} \quad (\text{A.9})$$

and

$$\kappa = \frac{C_s}{\ln 10} = \frac{C_s}{2.3} \quad (\text{A.10})$$

Critical state line

Increasing the shear stress on a soil specimen, will eventually lead to a state in which further shearing can occur without changes in volume, as shown in Figure A.2, known as Critical state condition. The critical state line (CSL) (Figures A.3a and b) is a representation of the critical state condition. The critical state line in $e-p'-q$ space is shown in Figure A.4 .

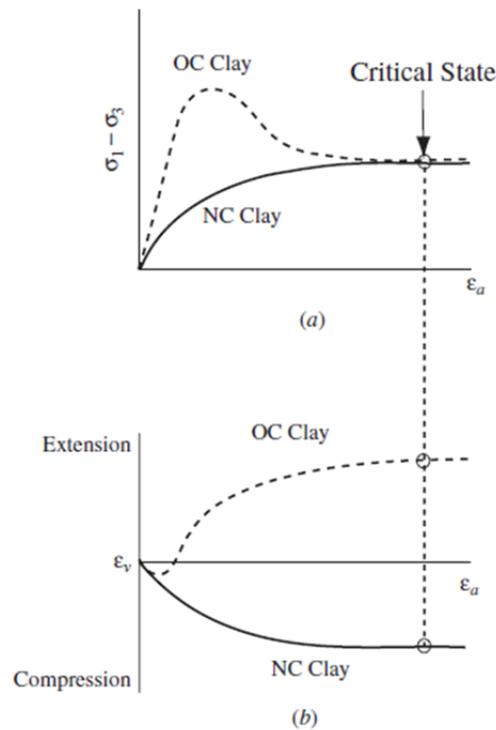


Figure A.2 Critical state definition (Helwany, 2007)

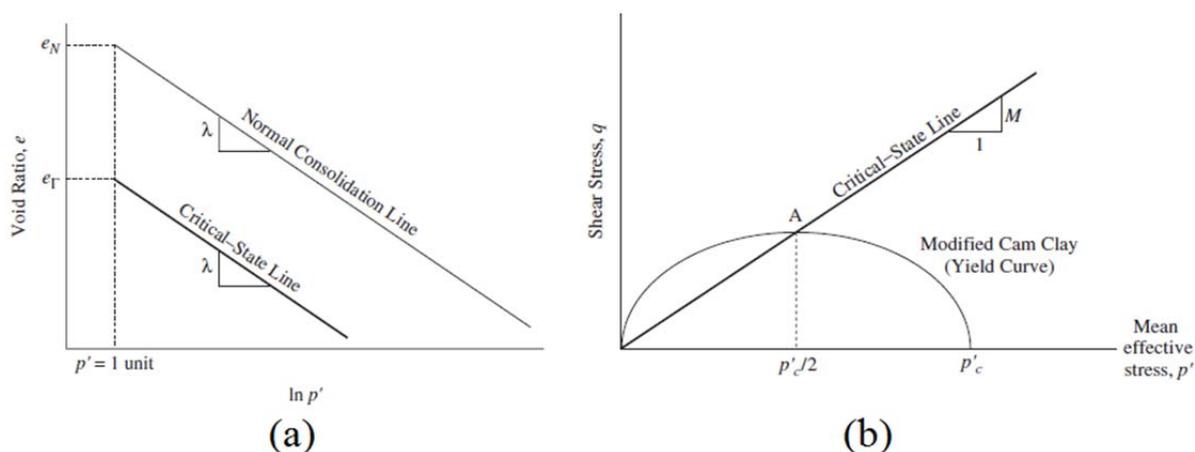


Figure A.3 (a) Normal consolidation and critical state lines in the e - $\ln p'$ plane (b) yield surface of a Cam-clay model in the q - p' plane (Helwany, 2007)

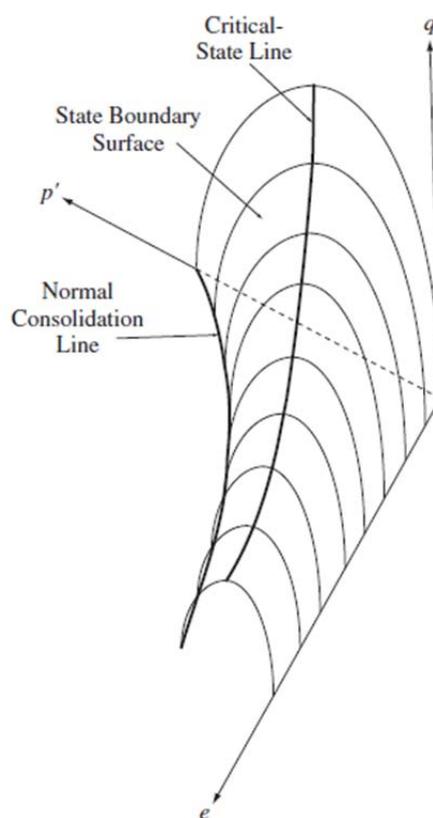


Figure A.4 State boundary surface of the Cam-clay model (Helwany, 2007)

Consolidated-drained (CD) or consolidated-undrained (CU) triaxial tests on representative soil specimens need to be conducted to obtain the critical state line. The critical state friction angle of the soil can be obtained from triaxial tests, by drawing the effective stress Mohr's

circle that represents a critical state condition. A straight-line tangent to the effective-stress Mohr's circle is drawn, which represents the effective-stress Mohr-Coulomb failure criterion. The slope of this line is the critical state friction angle ϕ' . The slope of the critical state line in the p' - q plane, M (Figure A.3b), can be calculated as

$$M = \frac{6 \sin \phi'}{3 - \sin \phi'} \quad (\text{A.11})$$

In reference to Figure A.3(b), the critical state line has the following equation in the p' - q plane:

$$q_f = Mp'_f \quad (\text{A.12})$$

where p'_f is the mean effective stress at failure and q_f is the shear stress at failure. This failure criterion bears the same meaning as the Mohr-Coulomb failure criterion

$$\tau_f = c' + \sigma' \tan \phi' \quad (\text{A.13})$$

where τ_f is the shear stress at failure and σ' is the effective normal stress, and c' is the cohesion.

The critical state line is parallel to the normal consolidation line in the e - $\ln p'$ plane, as shown in Figure A.3(a). The equation of the critical state line in this plane is given as

$$e_f = e_\Gamma - \lambda \ln p' \quad (\text{A.14})$$

where e_f is the void ratio at failure and e_Γ is the void ratio of the critical state line at $p' = 1 \text{ kPa}$. The parameters e_N and e_Γ are related by the equation

$$e_\Gamma = e_N - (\lambda - \kappa) \ln 2 \quad (\text{A.15})$$

Yield function

In the p' - q plane, the modified Cam-clay yield surface is an ellipse given by

$$\frac{q^2}{p'^2} + M^2 \left(1 - \frac{p'_c}{p'} \right) = 0 \quad (\text{A.16})$$

Figure A.3(b) shows an elliptical yield surface corresponding to a pre-consolidation pressure p'_c . The parameter p'_c controls the size of the yield surface and is different for each unloading-reloading line. The parameter p'_c is used to define the hardening behaviour of the soil. The soil behaviour is elastic until the stress state of the soil specimen (p', q) hits the yield surface. The soil behaves in a plastic manner after reaching the yield surface. Figure A.4 presents the yield surface in e - p' - q space, termed the state boundary surface.

Elastic response

The elastic response of the Modified Cam-clay model is defined by Young's modulus E , shear modulus G , Poisson's ratio ν , and bulk modulus K . These parameters are related by

$$E = 3K(1 - 2\nu) \quad (\text{A.17})$$

and

$$G = \frac{3K(1-2\nu)}{2(1+\nu)} \quad (\text{A.18})$$

The elastic behaviour of soil is nonlinear and stress-dependent. Therefore, the elastic moduli need be presented in incremental form.

For soils modelled by the Modified Cam-clay model, the bulk modulus K is stress-dependent. The bulk modulus depends on the mean effective stress p' , void ratio e_0 , and unloading-reloading line slope κ . The elastic behaviour is described by

$$K = \frac{(1+e_0)p'}{\kappa} \quad (\text{A.19})$$

Substituting Equation 39 into Equation 37 and 38 yields

$$E = \frac{3(1-2\nu)(1+e_0)p'}{\kappa} \quad (\text{A.20})$$

and

$$G = \frac{3(1-2\nu)(1+e_0)p'}{2(1+\nu)\kappa} \quad (\text{A.21})$$

Young's modulus E and shear modulus G are not constants. They are a function of the mean effective stress p' , void ratio e_0 , unloading-reloading line slope κ and Poisson's ratio ν . For simplicity, Poisson's ratio ν is commonly assumed to be constant.

Appendix B Additional FEA results

Additional results of the finite element analyses are presented in this appendix, including the pore pressure development, consolidation and shear stress-strain curves.

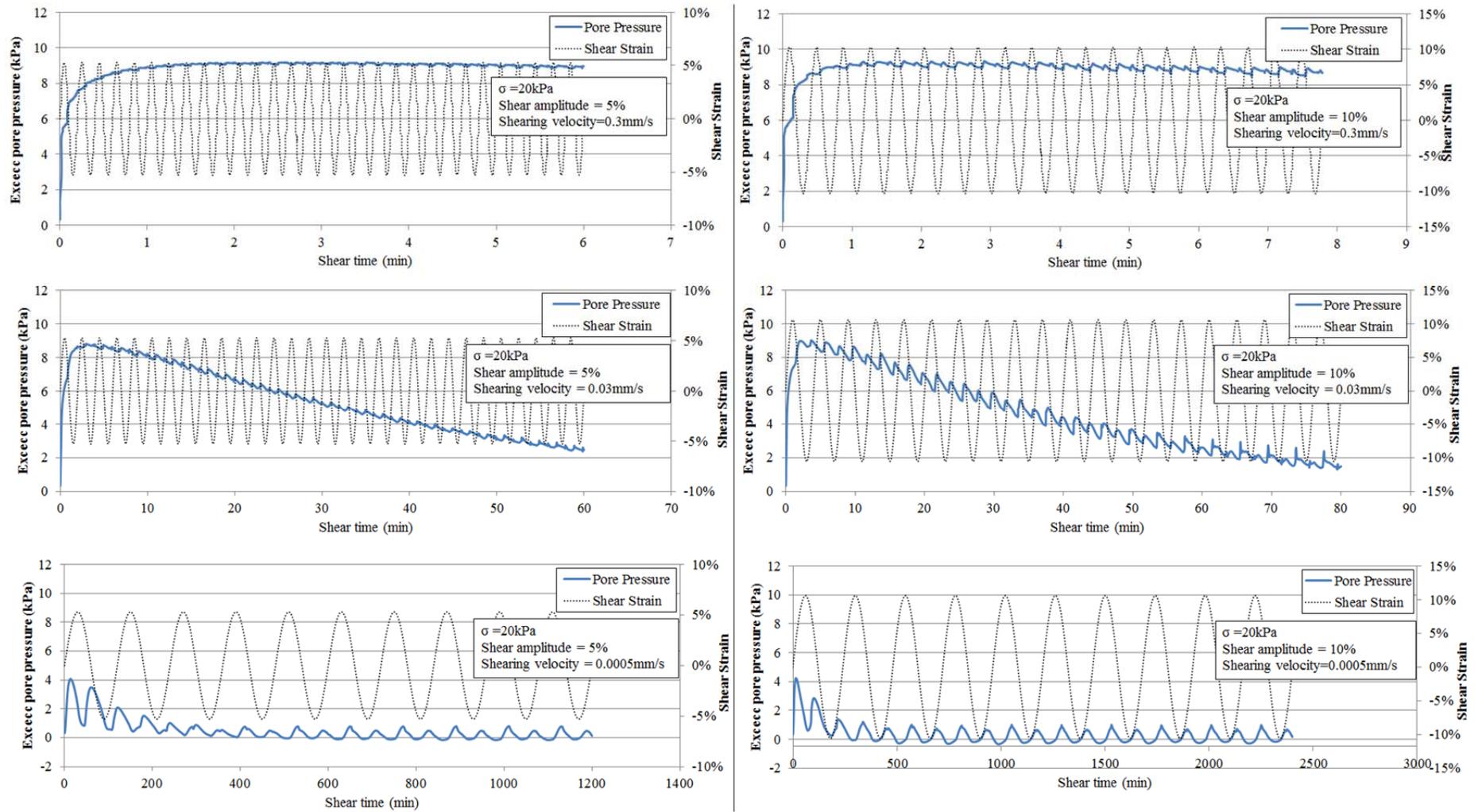


Figure A.5 Excess pore pressure development at cyclic shearing stage for applied vertical stress of 20kPa

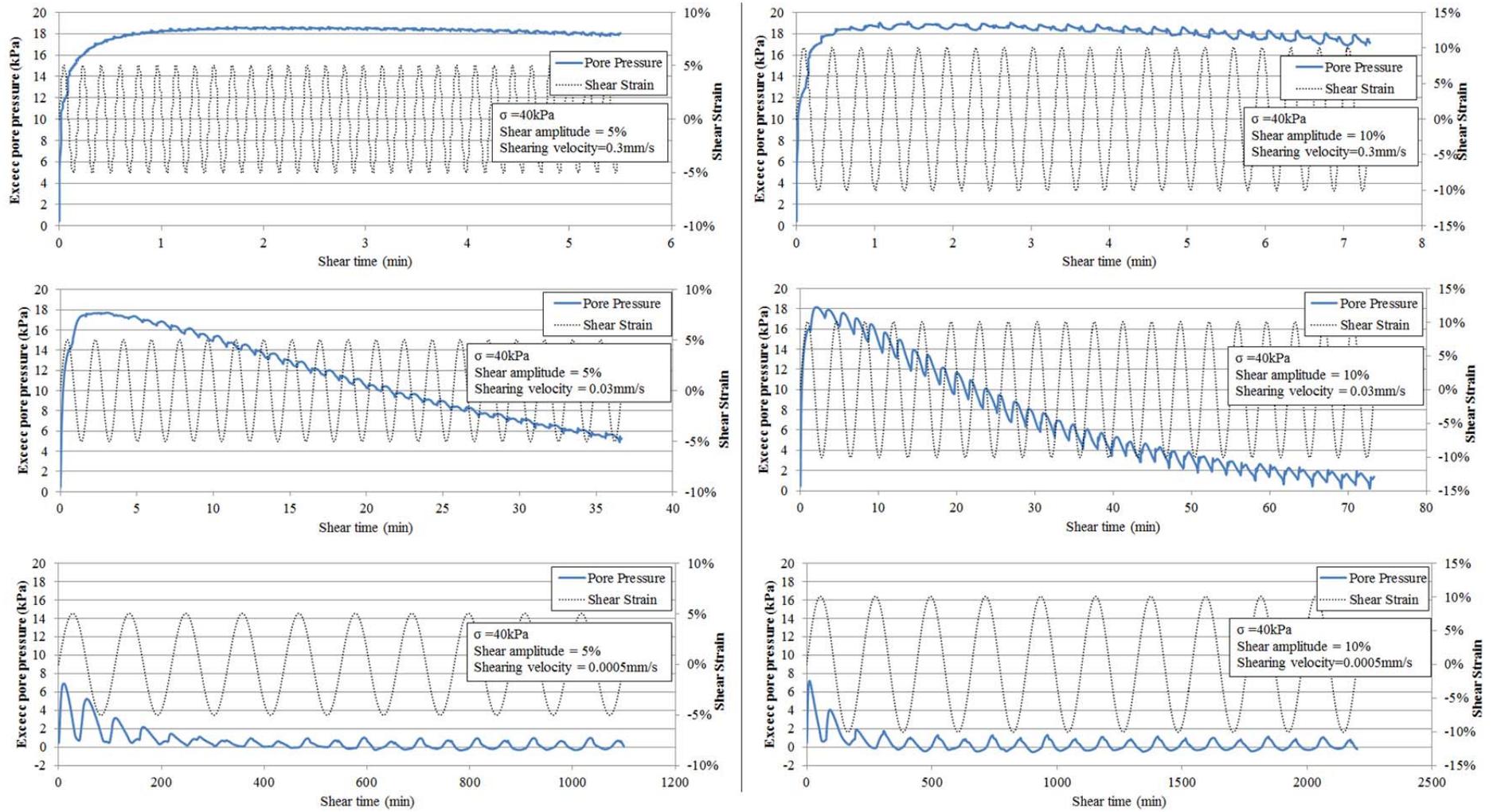


Figure A.6 Excess pore pressure development at cyclic shearing stage for applied vertical stress of 40kPa

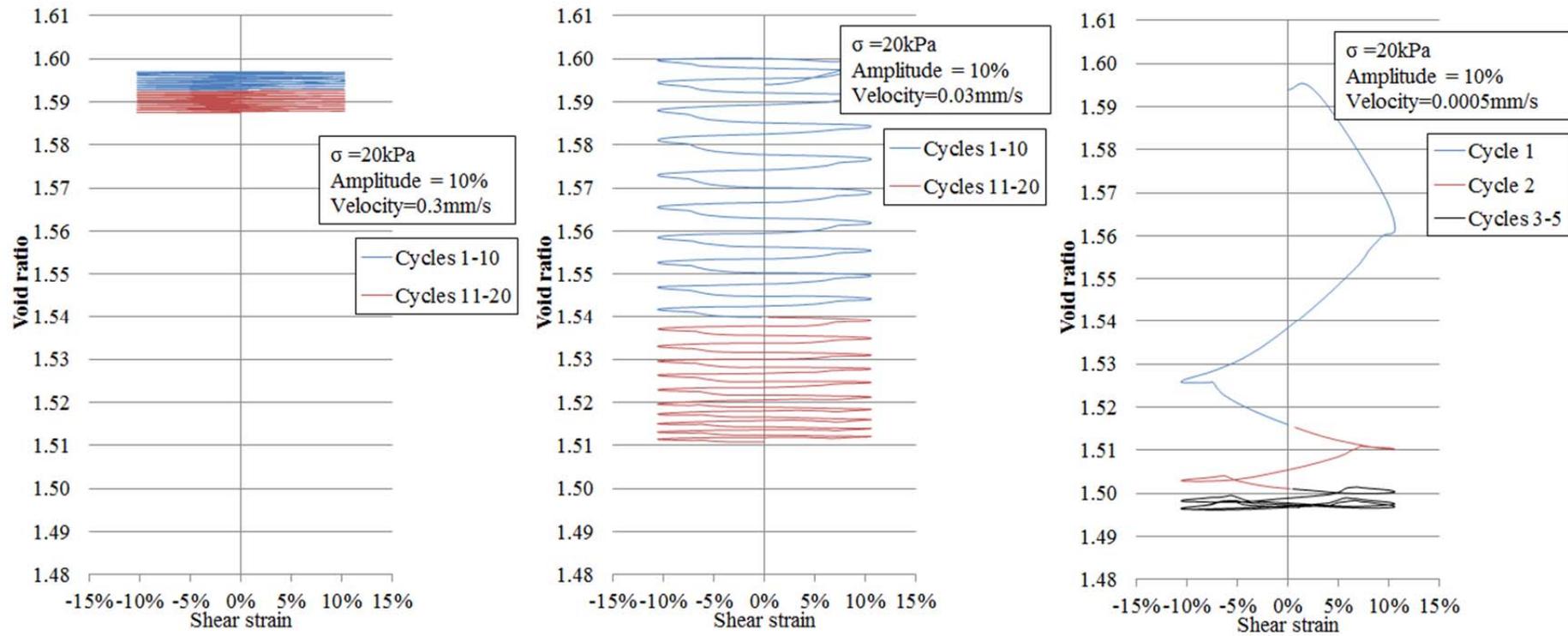


Figure A.7 Void ratio versus shear strain plots for applied vertical stress of 20kPa and shear amplitude of 10% of H

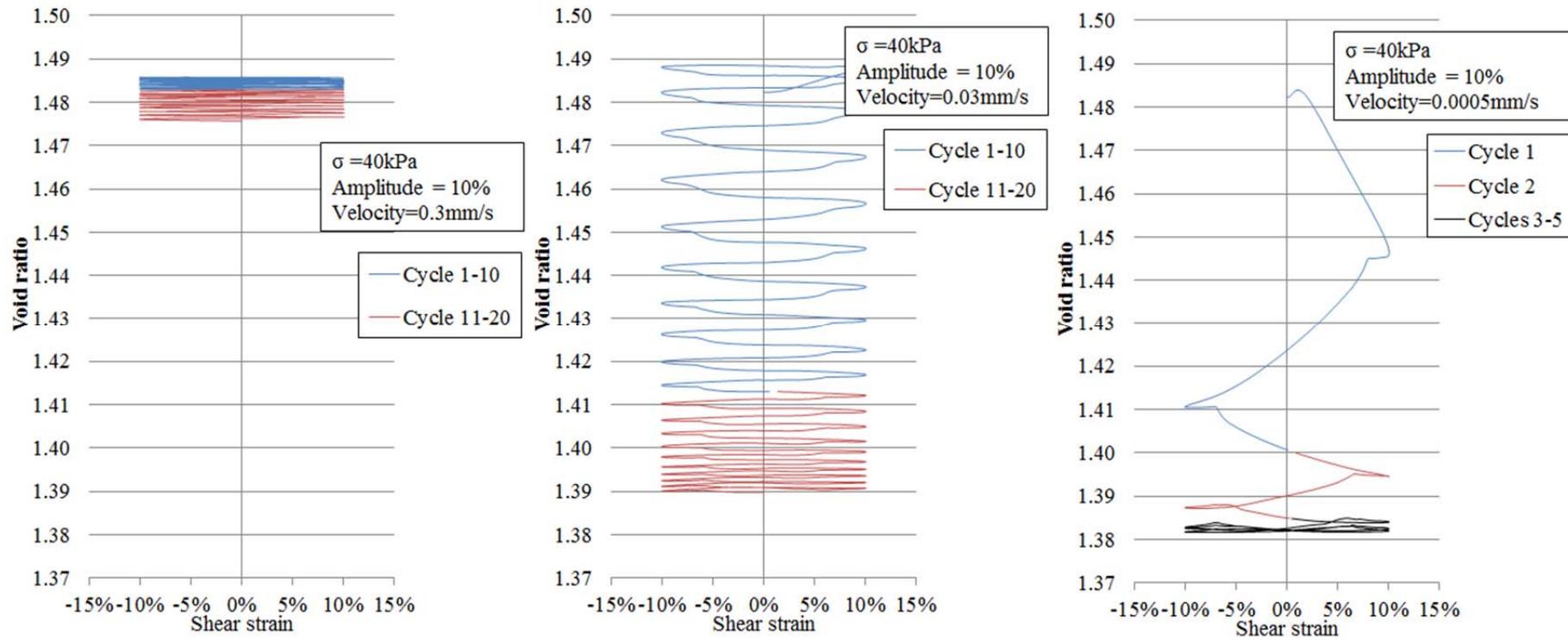


Figure A.8 Void ratio versus shear strain plots for applied vertical stress of 40kPa and shear amplitude of 10% of H

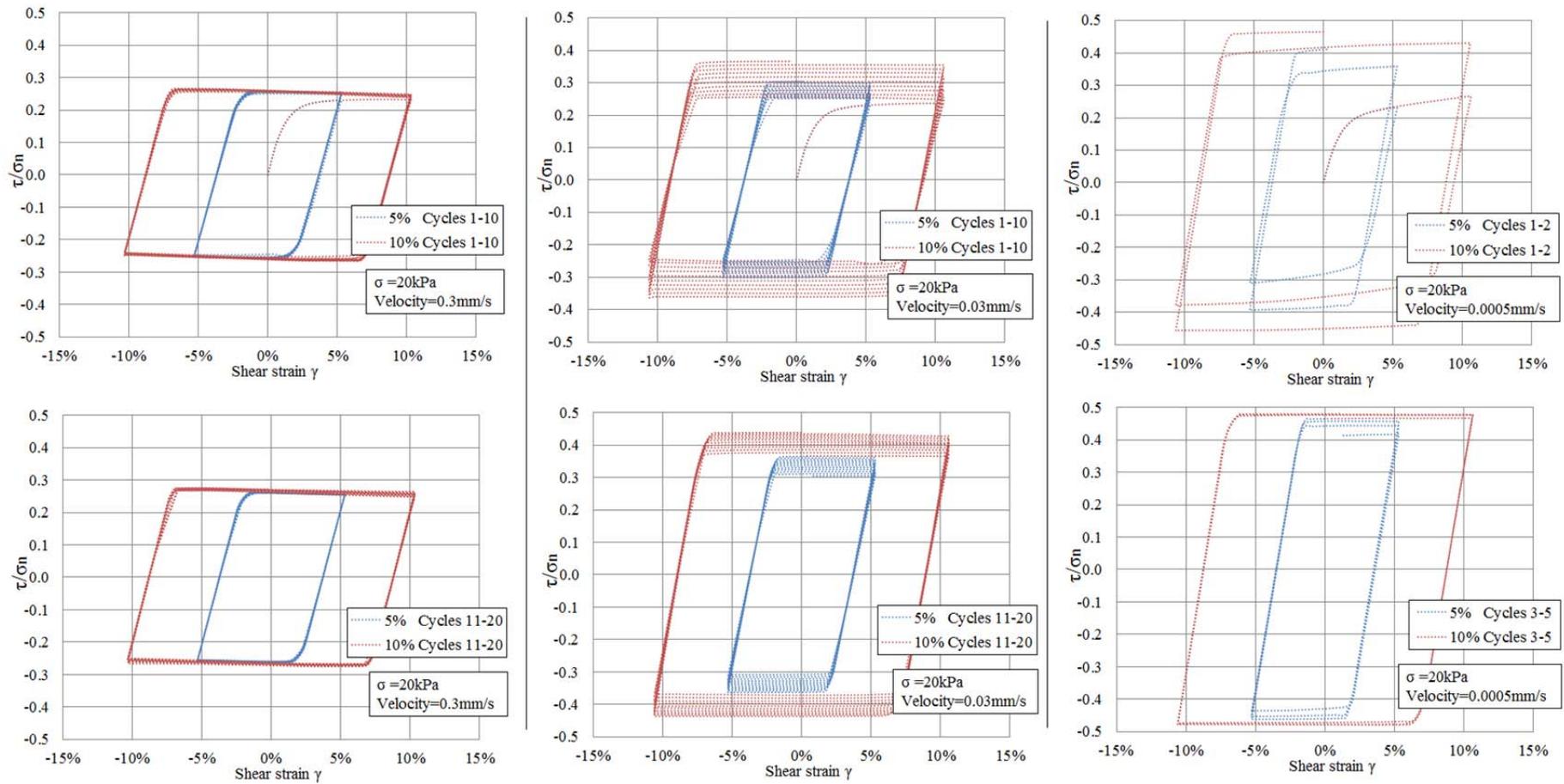


Figure A.9 Shear stress-strain curves for applied vertical stress of 20kPa

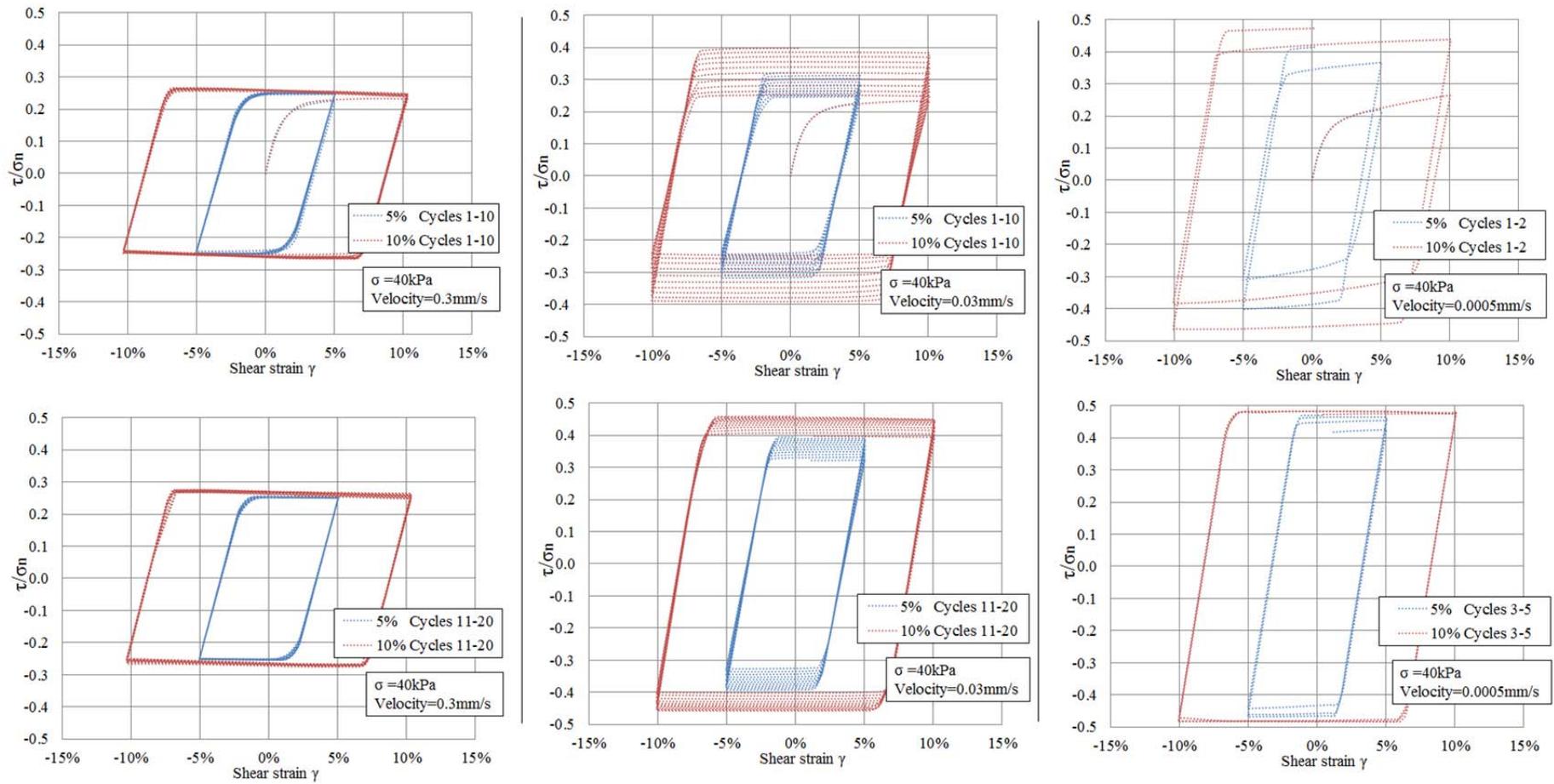


Figure A.10 Shear stress-strain curves for applied vertical stress of 40kPa

Appendix C sample .inp file of the finite element model

An example .inp file for the two dimensional finite element soil model developed is provided in this appendix. This .inp file is for an analysis with an applied vertical stress of 60kPa, shearing velocity of 0.03mm/s and shear amplitude of 10%. This file outlines how the model is constructed, how the soil properties are defined and how the loadings and boundary conditions are implemented.

```

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** Shearing velocity = 0.03mm/s
** Shearing amplitude = 10%
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** ASSEMBLY
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Appendices

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*Nset, nset=_PickedSet2, internal, generate
  1, 861, 1
*Elset, elset=_PickedSet2, internal, generate
  1, 800, 1
** Section: soil
** Solid Section, elset=_PickedSet2, material=camclay
,
*End Instance
**
*****
*****
*Nset, nset=bottom, instance=soil-1, generate
  1, 41, 1
*Elset, elset=bottom, instance=soil-1, generate
  1, 40, 1
*Nset, nset=top, instance=soil-1, generate
  821, 861, 1
*Elset, elset=top, instance=soil-1, generate
  761, 800, 1
*Nset, nset=sides, instance=soil-1
  1, 41, 42, 82, 83, 123, 124, 164, 165, 205, 206, 246, 247,
  287, 288, 328
  329, 369, 370, 410, 411, 451, 452, 492, 493, 533, 534, 574, 575,
  615, 616, 656
  657, 697, 698, 738, 739, 779, 780, 820, 821, 861
*Elset, elset=sides, instance=soil-1
  1, 40, 41, 80, 81, 120, 121, 160, 161, 200, 201, 240, 241,
  280, 281, 320
  321, 360, 361, 400, 401, 440, 441, 480, 481, 520, 521, 560, 561,
  600, 601, 640
  641, 680, 681, 720, 721, 760, 761, 800
*Nset, nset=soil, instance=soil-1, generate
  1, 861, 1
*Elset, elset=soil, instance=soil-1, generate
  1, 800, 1
*Nset, nset=_PickedSet9, internal, instance=soil-1, generate
  821, 861, 1
*Elset, elset=_PickedSet9, internal, instance=soil-1, generate
  761, 800, 1
*Elset, elset=_top_S3, internal, instance=soil-1, generate
  761, 800, 1
*Surface, type=ELEMENT, name=top
_top_S3, S3
*Nset, nset=bottom_PP_, internal, instance=soil-1, generate
  1, 41, 1
*Nset, nset=_PickedSet9_PP_, internal, instance=soil-1,
generate
  821, 861, 1
*Nset, nset=top_PP_, internal, instance=soil-1, generate
  821, 861, 1
*End Assembly
**
*****Defining Layer Sets
*Nset, nset=Layer-1, instance=soil-1
  821, 861
*Nset, nset=Layer-2, instance=soil-1
  780, 820
*Nset, nset=Layer-3, instance=soil-1
  739, 779
*Nset, nset=Layer-4, instance=soil-1
  698, 738
*Nset, nset=Layer-5, instance=soil-1
  657, 697
*Nset, nset=Layer-6, instance=soil-1
  616, 656
*Nset, nset=Layer-7, instance=soil-1
  575, 615
*Nset, nset=Layer-8, instance=soil-1
  534, 574
*Nset, nset=Layer-9, instance=soil-1
  493, 533
*Nset, nset=Layer-10, instance=soil-1
  452, 492
*Nset, nset=Layer-11, instance=soil-1
  411, 451
*Nset, nset=Layer-12, instance=soil-1
  370, 410
*Nset, nset=Layer-13, instance=soil-1
  329, 369
*Nset, nset=Layer-14, instance=soil-1
  288, 328
*Nset, nset=Layer-15, instance=soil-1
  247, 287
*Nset, nset=Layer-16, instance=soil-1
```


Appendices

```
*End Step
** -----
**
** STEP: Step-3
**
*Step, name=Step-3, nlgeom=YES, inc=10000, unsymm=YES,
AMPLITUDE=RAMP
*Soils, consolidation, end=ss, utol=100.
0.001, 172800., 1e-25, 500., 1e-15
** END=SS end when steady step is reached
** 0.001= initial increment size, 172800=period=48hrs, 1e-25=
min increment size,
** 500= max increment size
** End the consolidation when the pore pressure change is less
than 1e-15kPa
**
** BOUNDARY CONDITIONS
**
** Name: topporestep3 Type: Pore pressure
*Boundary
top_PP_, 8, 8, 0
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
** -----
**
** STEP: Step-4
**
*Step, name=Step-4, nlgeom=YES, inc=100000
*Soils, consolidation, end=PERIOD, utol=100.
0.001, 4240, 1e-25, 5.,
** 0.001=initial increment size, 4240=period, 1e-25=min inc
size, 5=max inc size
**
** Period = 212s, Number of Cycles = 20, Total Time=4240s
** BOUNDARY CONDITIONS
**
** Name: bottom Type: Displacement/Rotation
*Boundary, op=NEW
bottom, 2, 2
bottom, 1, 1
** Name: sides Type: Displacement/Rotation
*Boundary, op=NEW
** Name: topporestep3 Type: Pore pressure
** The pore pressure at top is set as 0kPa
*Boundary, op=NEW
top_PP_, 8, 8, 0
*****Define Top Shearing BCs
** Name: Shearing-Layer-1 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
top, 1, 1, 0.001
*****Define Linear Side BCs
** The following lines define the cyclic shear/horizntal
displacement of nodes at the vertical edges
** Name: Shearing-Layer-2 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-2, 1, 1, 0.00095
** Name: Shearing-Layer-3 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-3, 1, 1, 0.0009
** Name: Shearing-Layer-4 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-4, 1, 1, 0.00085
** Name: Shearing-Layer-5 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-5, 1, 1, 0.0008
** Name: Shearing-Layer-6 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-6, 1, 1, 0.00075
** Name: Shearing-Layer-7 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-7, 1, 1, 0.0007
** Name: Shearing-Layer-8 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-8, 1, 1, 0.00065
** Name: Shearing-Layer-9 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-9, 1, 1, 0.0006
** Name: Shearing-Layer-10 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-10, 1, 1, 0.00055
** Name: Shearing-Layer-11 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-11, 1, 1, 0.0005
** Name: Shearing-Layer-12 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-12, 1, 1, 0.00045
** Name: Shearing-Layer-13 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-13, 1, 1, 0.0004
** Name: Shearing-Layer-14 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-14, 1, 1, 0.00035
** Name: Shearing-Layer-15 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-15, 1, 1, 0.0003
** Name: Shearing-Layer-16 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-16, 1, 1, 0.00025
** Name: Shearing-Layer-17 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-17, 1, 1, 0.0002
** Name: Shearing-Layer-18 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-18, 1, 1, 0.00015
** Name: Shearing-Layer-19 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-19, 1, 1, 0.0001
** Name: Shearing-Layer-20 Type: Displacement/Rotation
*Boundary, op=NEW, amplitude=Cyclic_Shearing_Amp
Layer-20, 1, 1, 0.00005
*****
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field, variable=ALL
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
```

